

## SPS Adjusted Precalculus Scope & Sequence for 2020-21 SY (for students on track for AP Calculus BC)

The standards in **this course have been focused to address the content requirement for students to successfully enter Calculus or AP Calculus BC** in the following school year.

Precalculus Units of Study	Length of Unit
<b>Unit 1: How Functions Function</b>	<b>3 weeks</b>
<b>Unit 2: Exponential and Logarithmic Functions</b>	<b>3 weeks</b>
<b>Unit 3: Polynomial and Rational Functions</b>	<b>3 weeks</b>
<b>Unit 4: Trigonometry Fundamentals</b>	<b>5 weeks</b>
<b>Unit 5: Trigonometric Functions</b>	<b>6 weeks</b>
<b>Unit 6: Trigonometric Identities</b>	<b>2 weeks</b>
<b>Unit 8: Vectors</b>	<b>5 weeks</b>
<b>Unit 10: Sequences and Series</b>	<b>2 weeks</b>

### Priority standards and learning targets grouped by units.

Unit 1: How Functions Function	
Learning Targets	Standard(s) Aligned
Describe and compare key features of quadratic, exponentials, square root, and cube root functions.	F-IF.7 F-IF.9
Describe the transformation of a parent function from a graph or equation.	F-BF.3
Graph piece-wise functions and describe key features.	F-IF.7b
Compose functions to create new functions.	F-BF.1C
Identify inverse functions through composition.	F-BF.4b

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Unit 2: Exponential and Logarithmic Functions	
Learning Targets	Standard(s) Aligned
Prove through composition that logarithmic and exponential functions are inverses.	F-BF.4b
Graph logarithmic functions, showing intercepts and end behavior.	F-IF.7e
Derive the properties of logarithms from exponential properties.	ALT 2.1
Apply the properties of logarithms to rewrite logarithmic and exponential expressions.	ALT 2.1 ALT 2.2
Solve logarithmic and exponential equations using the properties of logarithms.	ALT 2.4
Model a real-world situation with a logarithmic function.	ALT 2.5

Unit 3: Polynomial and Rational Functions	
Learning Targets	Standard(s) Aligned
Graph polynomial functions and identify key features.	F-IF.7c
Solve for the zeros of a polynomial through factorization.	A-APR.3
Graph the parent rational function, identify key features, and describe transformations of the parent functions.	F-IF.7d
Rewrite rational functions and examine their graphs to reveal key features, including intercepts, <i>horizontal</i> and vertical asymptotes and end behavior.	A-APR.6
Rewrite rational functions and examine their graphs to reveal key features, including intercepts, vertical and <i>slant</i> asymptotes and end behavior.	A-APR.6 F-IF.7d

Unit 4: Trigonometric Fundamentals	
Learning Targets	Standard(s) Aligned
Define radian measure and identify corresponding radian and degree measures.	F-TF.1
Define sine and cosine as coordinates of points along the unit circle in relation to radian measure.	F-TF.3
Identify sine and cosine of special angles ( $\frac{\pi}{3}$ , $\frac{\pi}{4}$ , $\frac{\pi}{6}$ , and those angles $\pm k\frac{\pi}{2}$ ) on the unit circle using special right triangles and Pythagorean theorem.	F-TF.3
Locate any point on the unit circle given the radian measure by finding sine and cosine using technology.	F-TF.3
Given a right triangle in space, identify the values of the 6 trigonometric ratios and describe how they are related.	G-SRT.8
Describe how the six trig functions change as a ray rotates around the unit circle based on geometric relationships.	ALT 4.1
Calculate the exact and estimated values of all six trig functions for any angle given in degrees or radians.	ALT 4.1

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<b>Unit 5: Trigonometric Functions</b>	
<b>Learning Targets</b>	<b>Standard(s) Aligned</b>
Graph sine and cosine parent functions in radians.	F-IF.7e
Describe and graph transformations of sine and cosine.	F-IF.7e F-BF.3
Write the equation of a transformed sine or cosine function from a graph.	ALT 5.1
Graph the tangent, cotangent, secant, and cosecant functions and identify key features.	ALT 5.2
Graph the inverse trigonometric relationships; restrict domains to create inverse trigonometric functions.	ALT 5.3
Solve for possible angle measurements using inverse trig operations.	F-TF.7
Model periodic phenomena and/or circular motion with sine and cosine functions.	F-TF.5

<b>Unit 6: Trigonometric Identities</b>	
<b>Learning Targets</b>	<b>Standard(s) Aligned</b>
Derive and apply the Pythagorean identities.	F-TF.8
Use basic trigonometric identities to complete proofs of other trigonometric identities.	ALT 6.2

<b>Unit 8: Vectors</b>	
<b>Learning Target</b>	<b>Standard(s) Aligned</b>
Define and represent a vector graphically and algebraically.	N-VM.1 N-VM.2
Add and subtract vectors, and multiply vectors by a scalar.	N-VM.4 N-VM.5
Solve problems involving velocity and other quantities that can be represented by vectors.	N-VM.3
Write the parametric form of rectangular coordinates.	ALT 8.2
Model and graph quantities involving vectors using parametric equations.	ALT 8.2
Describe the features of a polar coordinate plane.	
Convert between rectangular and polar coordinates.	
Graph polar equations.	

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Unit 10: Sequences and Series	
Learning Target	Standard(s) Aligned
Identify arithmetic and geometric sequences.	F-BF.2
Write explicit and recursive equations for arithmetic and geometric sequences.	F-BF.2
Calculate the sum of a finite arithmetic series.	ALT 10.1
Calculate the sum of a finite geometric series.	A-SSE.4

## Common Core Standards and SPS Additional Learning Targets in Precalculus:

Conceptual Category: Number and Quantity
<b>Standards</b>
<p><b>Vector and Matrix Quantities</b></p> <p><b>N-VM.1 (+)</b> Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., <math>v</math>, <math> v </math>, <math>  v  </math>, <math>v</math>).</p> <p><b>N-VM.2 (+)</b> Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p> <p><b>N-VM.3 (+)</b> Solve problems involving velocity and other quantities that can be represented by vectors.</p> <p><b>N-VM.4 (+)</b> Add and subtract vectors.</p> <p style="padding-left: 20px;"><b>A. (+)</b> Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p style="padding-left: 20px;"><b>B. (+)</b> Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p style="padding-left: 20px;"><b>C. (+)</b> Understand vector subtraction <math>v - w</math> as <math>v + (-w)</math>, where <math>-w</math> is the additive inverse of <math>w</math>, with the same magnitude as <math>w</math> and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</p> <p><b>N-VM.5 (+)</b> Multiply a vector by a scalar.</p> <p style="padding-left: 20px;"><b>A. (+)</b> Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as <math>c(v_x, v_y) = (cv_x, cv_y)</math>.</p> <p style="padding-left: 20px;"><b>B. (+)</b> Compute the magnitude of a scalar multiple <math>cv</math> using <math>  cv   =  c v</math>. Compute the direction of <math>cv</math> knowing that when <math> c v \neq 0</math>, the direction of <math>cv</math> is either along <math>v</math> (for <math>c &gt; 0</math>) or against <math>v</math> (for <math>c &lt; 0</math>).</p>

Conceptual Category: Algebra
<b>Standards</b>
<p><b>Seeing Structure in Expressions</b></p> <p><b>A-SSE.4</b> Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.</p> <p><b>Arithmetic with Polynomials and Rational Expressions</b></p> <p><b>A-APR.3</b> Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.</p> <p><b>A-APR.6</b> Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p>

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Conceptual Category: Functions
Standards
<p><b>Interpreting Functions</b></p> <p><b>F-IF.7</b> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <ul style="list-style-type: none"> <li>A. Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>B. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> <li>C. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> <li>D. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</li> <li>E. Graph exponential and logarithmic functions, showing intercepts and end behavior and graph trigonometric functions, showing period, midline, and amplitude.</li> </ul> <p><b>F-IF.9</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p>
<p><b>Building Functions</b></p> <p><b>F-BF.1</b> Write a function that describes a relationship between two quantities.</p> <ul style="list-style-type: none"> <li>C. (+) Compose functions.</li> </ul> <p><b>F-BF.2</b> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p> <p><b>F-BF.3</b> Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs.</p> <p><b>F-BF.4</b> Find inverse functions.</p> <ul style="list-style-type: none"> <li>A. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse.</li> <li>B. (+) Verify by composition that one function is the inverse of another.</li> <li>C. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</li> </ul> <p><b>F-BF.5</b> (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p>
<p><b>Trigonometric Functions</b></p> <p><b>F-TF.1</b> Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p><b>F-TF.3</b> (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\frac{\pi}{3}</math>, <math>\frac{\pi}{4}</math>, and <math>\frac{\pi}{6}</math>, and use the unit circle to express the values of sine, cosine, and tangent for <math>\pi - x</math>, <math>\pi + x</math>, and <math>2\pi - x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</p> <p><b>F-TF.5</b> Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.</p> <p><b>F-TF.7</b> (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.</p>

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**F-TF.8** Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

### Conceptual Category: Geometry

#### Standards

#### Similarity, Right Triangles, and Trigonometry

**G-SRT.8** Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.

### Seattle Public Schools Additional Learning Targets (ALT)

#### Learning Targets

#### Unit 2: Exponential and Logarithmic Functions

**ALT 2.1** Use the properties of logarithms to transform logarithmic expressions.

**ALT 2.2** Use the definition of a logarithm to translate between exponential and logarithmic form, using any base.

**ALT 2.3** Use the change of base formula to evaluate logarithms that have bases other than 10 or  $e$ .

**ALT 2.4** Use the definition of a logarithm, properties of logarithms, and the change of base formula to solve equations where the unknown value is an exponent.

**ALT 2.5** Write and use logarithmic functions in any base to model real-world phenomena.

#### Unit 4: Trigonometric Fundamentals

**ALT 4.1** Use special triangles to determine geometrically the values of secant, cosecant, cotangent for  $\frac{\pi}{3}$ ,  $\frac{\pi}{4}$ , and  $\frac{\pi}{6}$ , and use the unit circle to express the values of secant, cosecant, cotangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.

#### Unit 5: Trigonometric Functions

**ALT 5.1** Given a sinusoidal graph, write the related trigonometric function.

**ALT 5.2** Graph all trigonometric functions –  $f(x) = \sin, \cos, \tan, \sec, \csc, \text{ or } \cot(x)$ .

**ALT 5.3** Graph inverse trigonometric functions.

#### Unit 6: Trigonometric Identities

**ALT 6.2** Prove that two trigonometric expressions are equivalent using identities

#### Unit 10: Probability

**ALT 10.1** Derive the formula for the sum of a finite arithmetic series and use the formula to solve problems.