# Contents

The *Discovering Geometry* Approach to Learning ........................................... v  
Working with Your Student ................................................................................... ix  
Overview of Topics in *Discovering Geometry* ................................................... xii 
Chapter 0: Geometric Art ...................................................................................... 1  
   Content Summary .............................................................................................. 1  
   Summary Problem ............................................................................................ 2  
   Chapter 0 Review Exercises .......................................................................... 3  
   Solutions to Chapter 0 Review Exercises ...................................................... 4  
Chapter 1: Introducing Geometry ...................................................................... 5  
   Content Summary .............................................................................................. 5  
   Summary Problem ............................................................................................ 6  
   Chapter 1 Review Exercises .......................................................................... 7  
   Solutions to Chapter 1 Review Exercises ...................................................... 8  
Chapter 2: Reasoning in Geometry ................................................................... 9  
   Content Summary .............................................................................................. 9  
   Summary Problem ............................................................................................ 10  
   Chapter 2 Review Exercises .......................................................................... 11  
   Solutions to Chapter 2 Review Exercises ...................................................... 12  
Chapter 3: Using Tools of Geometry ................................................................ 13  
   Content Summary .............................................................................................. 13  
   Summary Problem ............................................................................................ 14  
   Chapter 3 Review Exercises .......................................................................... 15  
   Solutions to Chapter 3 Review Exercises ...................................................... 16  
Chapter 4: Discovering and Proving Triangle Properties ................................ 17  
   Content Summary .............................................................................................. 17  
   Summary Problem ............................................................................................ 18  
   Chapter 4 Review Exercises .......................................................................... 19  
   Solutions to Chapter 4 Review Exercises ...................................................... 20  
Chapter 5: Discovering and Proving Polygon Properties ................................ 21  
   Content Summary .............................................................................................. 21  
   Summary Problem ............................................................................................ 21  
   Chapter 5 Review Exercises .......................................................................... 23  
   Solutions to Chapter 5 Review Exercises ...................................................... 24
Discovering Geometry: An Investigative Approach covers the topics you would expect from a geometry course, but the teaching style as well as the learning experience might be different from what you remember from your own high school geometry course.

In the past, and perhaps in your own school experience, geometry was about memorizing a set of postulates and proving a long list of theorems, not about understanding properties of shapes or solving practical problems. For example, you might recognize this scenario: After going over the homework, your teacher introduced a new theorem, went over a proof of the theorem, and showed how to use the theorem to solve a problem. Then you worked alone with paper and pencil and practiced solving problems of the same type. For homework, you worked on more problems of the same type and tried to write a proof related to the new theorem. The next day, the class went through that same process with a new theorem. At some point, you took a test. You had to remember the theorems and figure out what theorem to use for each problem. If you did well on all the tests, you were “good at math.” If you didn’t do well, you might have thought you “just weren’t good at it.”

Many students cannot succeed in such an environment. When learning focuses on mechanical manipulations, students are limited in their understanding. They don’t know when to apply a particular problem-solving strategy. They don’t come away from their math course with a set of ideas that weave together into “the big picture.” They doubt that mathematics will be relevant to their careers and they don’t see what others like about it. Even students who pass are reluctant to continue on in mathematics. Some develop “math phobia”—the fear of math—and avoid courses in science or business that require math. Ultimately, their fear limits their career choices and earning potential.

But all students can learn math better, have a good time doing it, and come away with an appreciation of its value as a tool for science, business, and everyday life. Discovering Geometry is a program that helps all students reach a deep understanding of math by encouraging them to investigate interesting problems in cooperative groups, use technology where appropriate, and practice using problem-solving skills to make tough problems manageable.

All Students Benefit

Michael Serra, the author of Discovering Geometry, taught geometry for 32 years. He knows from his own teaching experience that all students can experience more success in mathematics. When the focus is on understanding concepts and reasoning strategies instead of just memorizing formulas and theorems, students who have previously been identified as having concentration, attention, and memory issues can be more successful. Passive or reluctant students will learn to communicate better.

To say that all students can learn geometry does not mean the course has been watered down. In fact, even very successful math students will find they are challenged, learn more, and remember longer with the Discovering Geometry Approach to Learning.
approach. That’s because the concepts and methods are not isolated from real-world applications or from previously learned ideas or from information they are getting in other classes. The mathematics that students study is closer to what is needed by both students seeking employment after high school and students preparing to attend college.

**Deep Understanding Is Important**

In your own math classes, you might have been told: “Just do it—don’t ask why.” But there are logical reasons behind mathematical methods and ideas, and the people who understand these reasons succeed at math and, ultimately, at science and business. *Discovering Geometry* helps more students understand these reasons. Because the concepts make sense to students, students remember the methods (or reinvent them if they’ve forgotten them) and can apply them to new problems. To help develop that kind of flexible understanding, *Discovering Geometry* offers a more visual approach, with clearer and more frequent illustrations and photographs, and the use of text captions to lead students through examples. *Discovering Geometry* also acknowledges the need for a gradual development of mathematical ideas. Students are helped to see where the text is leading, and full-blown explanations are delayed until all the groundwork is laid. Once a topic has been made part of what students are expected to know, it is reviewed and referred to again whenever appropriate. Understanding the math can make the math more fun for students, will bolster their pride and confidence, and ultimately increase the chance that students will use math in their lives.

**Students Learn Better in Cooperative Groups**

Students are not expected to do all this learning by themselves. Students often make sense of mathematical ideas best in interaction with other people, using informal language. They get better at explaining their thinking when they think out loud, and they get ideas from others. They learn that nothing bad happens if they make mistakes or misapply a procedure and that trial and error is also a respected strategy. This helps quiet or insecure students learn to contribute.

Group work helps students learn better, and it also teaches essential teamwork skills. When students are working in groups, the teacher circulates and observes, poses questions, and intervenes when necessary to assist. He or she works as a guide to student groups, monitoring the back-and-forth, modeling good communication, and drawing out clues that indicate whether students are confused or on the right track. In their groups, students will be asked to demonstrate their understanding both orally and in writing.

**Investigation Is Motivating**

Some students learn better by seeing, some by hearing, some by reading, and some by using geometry tools to see for themselves how geometry works. An explanation that makes sense to one student might not make sense to another. These different “learning styles” are addressed by the investigations in *Discovering Geometry*. In this program, students use tools—protractors, rulers, compasses and straightedges, “patty paper” (squares of tracing paper), and perhaps geometry software—to learn the properties of shapes. They use drawings and measurements to make observations and form hypotheses, or “conjectures.” Similar to the way in which scientists work, they examine their conjectures and try to determine
whether these conjectures are always true. They look for exceptions to the rule, or they try to prove that the conjecture is always true without exception. As they carry out their investigations, students keep a notebook that includes a list of definitions, a list of conjectures, and answers to Investigations and homework exercises.

The teacher might have students work in groups on an investigation, and later lead a whole-class discussion. Each student develops his or her own understanding and benefits from sharing ideas and suggestions offered by others. Students learn that there are many approaches to solving problems. They also learn that they are individually responsible for describing orally or in writing what they have learned.

**Problem Solving Is Important**

In life, we all need to be good at solving new problems. This is an important job skill and career asset as well: People who “think outside the box” to solve problems at work move up faster and are seen as leaders. To help prepare students to use math in their lives, many exercises in Discovering Geometry pose problems that students haven’t already been told how to solve. They learn to brainstorm, consider subproblems, come at a problem from a unique angle, and make diagrams and models. In this way, they learn problem-solving skills rather than learning how to solve only particular types of problems.

Because most students are more interested in class if the problems they investigate are related to the real world, many of the homework exercises involve problems that students might see in their lives outside school. Some use very familiar scenarios, and others are career-oriented.

Application exercises may relate to agriculture (for example, Exercise 12 on p. 451, Exercise 16 on p. 468, Exercise 10 on p. 564), business (for example, Exercise 1 on p. 435, Exercise 46 on p. 475, Exercise 8 on p. 611), construction and maintenance (for example, Exercise 16 on p. 261, Exercise 20 on p. 432, Exercise 19 on p. 574), cooking (for example, Exercise 13 on p. 455, Exercise 22 on p. 574, Exercise 24 on p. 629), architecture (for example, Exercise 17 on p. 273, Exercise 11 on p. 352, Exercise 9 on p. 564), technology (for example, Exercise 3 on p. 435, Exercise 21 on p. 514, Exercise 4 on p. 668), or sports and recreation (for example, Exercise 18 on p. 219, Exercise 15 on p. 451, Exercise 5 on p. 668).

**Using Technology Helps**

Computers and calculators surround us, and students will use them at work, sometimes with custom-designed software, so working with these tools now teaches students skills that will be useful later on. Whether you are computer literate yourself or strictly “low-tech,” your student is probably fascinated with technology, and using technology in class will help keep your student interested.

Technology is not used as a substitute for learning basic concepts. When used appropriately, technology can make mathematics more visual, more logical, and more fun. Most importantly, technology tools allow students to investigate many more situations and examples than they can explore by using pencil and paper. Getting fast results on numerous examples helps students see patterns, form generalizations, and test conclusions. That leads to a deeper understanding of concepts and a greater willingness to explore further and tackle larger problems.
If your student’s teacher does not have access to technology or doesn’t have access to enough computers, Dynamic Geometry Explorations are available online at flourishkh.com. These interactive sketches can be used by teacher for demonstrations or by students for a rich and highly visual learning experience.

**Review Is Essential**

Students apply their new skills regularly with the exercises in the student text. Each lesson also has Review exercises so that students retain and extend their learning from previous lessons. For further practice, your student’s teacher has probably received a copy of *Discovering Geometry: Practice Your Skills*. You can access these worksheets online at flourishkh.com.

Algebra review is important as well. *Discovering Geometry* helps students see connections between geometry and algebra by integrating the review of algebra skills into every set of exercises. It also devotes one lesson in each chapter specifically to reviewing and practicing a key algebra topic. These “Using Your Algebra Skills” lessons may be assigned by the teacher, or you can use them with your student at home.

Finally, Chapter Reviews and Mixed Reviews help students prepare for chapter tests and end-of-term exams. Answers to these review sections are in the back of the book so that you can help your student check for understanding and determine whether some areas need more work.
Working with Your Student

Begin by taking stock of how your student uses his or her after-school time. Evaluate whether there is a suitable place with good light to make homework a comfortable activity, and whether distractions in the homework environment are manageable. Your support and praise are as important to your student’s success as the teacher’s guidance and the quality of the learning materials.

Use this guide in conjunction with the Discovering Geometry textbook. Refer to the notes on individual chapters. References are made to specific examples and exercises in the text. In addition, find out what resources your student has at school and what he or she can access online or from home.

Your Own Experience with Math Is a Big Influence

Did you do well in math when you were in school? If math was hard for you, you might actually find it easier to help your student, because you’ll be especially sympathetic. You’ve probably also developed some practical understanding since you left school. The important thing is to avoid passing on negative ideas about math. You have the chance to help your student have a better attitude toward math. Your message must be “mathematics is important for everyone.” To be successful in our society, everyone must be able to recognize when a situation needs a mathematical solution, to tell what quantities are involved, and to understand how to work toward a solution. Your student has the benefit of a better approach and better materials than you may have had.

What if you’re confident in your math skills? You will have to be careful not to dominate your student’s learning. It’s sometimes very hard to resist explaining an idea or giving an answer you understand, but holding back is necessary if your student is to remember the idea and ultimately become an independent learner. Praise all your student’s honest efforts and support his or her attempts to explain, question, or break down the problem.

No matter how comfortable you are with math, you can help your student reach the goals of the Discovering Geometry approach and learn geometry. Try to establish two habits when you work with your student.

First, be a student to your student. Keep asking him or her for explanations. Ask questions as if you were the student trying to learn. No matter how well you understand things yourself, asking “Why does that work?” is better than saying “Here’s how to do that.”

Second, be curious and enthusiastic. Offer comments like “I haven’t seen this idea before, but it seems interesting” rather than “It’s beyond me!” or “This isn’t important.” Ask what happened in class, ask what your student contributed and how well he or she understood, and be curious about the homework. Showing this kind of interest says that you expect your student to be actively involved in class and to work on homework every day.
**Use Tried and True Reasoning Strategies**

Some classic reasoning strategies can help your student, especially when writing proofs, and you can assist him or her to use them.

1. **Draw a labeled diagram and mark what you know.** This is very helpful for real-world problems, or problems that have geometric figures or a coordinate grid. Be sure your student is the one doing the drawing. You can coach, ask questions, or make suggestions: “Why not draw a line for the wall?” “Where is the person standing?” Encourage your student to *label parts of the diagram* with quantities that represent distance or other measurements, and to use arrows for motion.

2. **Represent a situation algebraically.** When trying to find a missing quantity, it is helpful to list the known quantities and think about how to find the one that is missing. Your student can assign a variable (letter name) to each quantity—for instance, use $A$ for area of a circle and $r$ for the radius. He or she can use these variables to write an equation that shows the relationship between these quantities and solve for the unknown. The Table of Symbols at the end of the Glossary can help identify the correct symbols.

3. **Apply previous conjectures and definitions.** Your student has a list of conjectures from investigations done in class, as well as a list of definitions. Looking back over these will help your student remember what was learned in class and may give some insights into the homework exercises, especially the proofs. Asking about what was done in class might help jog your student’s memory. Play the role of a student and let him or her explain to you what the lesson was about and what was learned. Then ask how that might relate to the problem you are trying to solve.

4. **Break a problem into parts.** Your student should work on one stage of a problem at a time. This makes a difficult problem more manageable. Or, he or she can solve an easier related problem. This might help your student recognize a process that he or she remembers and understands. See if your student can explain where he or she “got stuck”—this will make it easier to ask for help later. Be sure to praise success on easier problems and partial answers to demonstrate your support and let your student know that he or she has some level of ability and achievement.

5. **Add an auxiliary line.** When working with geometric figures, your student can draw an auxiliary line, or “helping line,” if it helps to solve the problem. For example, he or she can draw the altitude of a triangle perpendicular to a base. In the case of an isosceles triangle, the altitude to the base forms two congruent triangles. Your student can use the congruent triangles to prove that certain segments and angles are congruent. Or, he or she can draw a radius of a circle if it is not already on the figure. All radii of a circle are congruent, and your student can use this fact to make certain observations. Sometimes it may help to draw a diagonal of a quadrilateral to form two triangles.

6. **Work backward.** Start at the end of a series of steps and see how it feels to work toward the beginning. This is a good way to start developing a plan for writing a proof, or to check whether a guessed answer is right and to understand why it was a good guess. **Guess-and-check** is a good strategy in itself if it helps your student gets closer and closer to the right answer in successive steps.
Learn About and Use Other Resources

If your student continues to have homework trouble even after you have tried to help, you can guide him or her to list questions for the teacher. This list will help the teacher know whether the student feels that he or she basically understood the lesson and is simply stuck on a single problem, or whether the student feels completely off track and hasn’t understood the lesson or even the last several lessons. Are there particular concepts that your student doesn’t understand? Is there an example in the book that he or she cannot follow? Helping your student script questions to the teacher will reduce anxiety or shyness about asking for help. If, finally, your student feels unable to ask for assistance, you should intervene with a note or call to the teacher.

Your student can do projects using The Geometer’s Sketchpad Dynamic Geometry® software and Fathom Dynamic Data™ software. Discounted student editions of these software programs are available at keycurriculum.com. You can also download from flourishkh.com Condensed Lessons (in English or Spanish) in case your student has missed class or is working with a tutor, and Practice Your Skills for extra reinforcement.

If you have Internet access, you can enrich your student’s experience by having your student follow Web links and view the Dynamic Geometry Explorations available for Discovering Geometry. You can also download the practice worksheets. Find these at flourishkh.com.

If the teacher has registered, you can access the online version of Discovering Geometry. Discovering Geometry Online is a service that provides students with access to all the content of their printed textbook page by page in an easy-to-use format. The online textbook has an interactive glossary and an index, and direct links to the chapter-specific resources just mentioned.

Discovering Geometry has been designed with an investigative approach to engage your student in doing mathematics—understanding, learning, remembering, and applying geometry skills. With your student’s growing sense of responsibility for his or her own learning, a teacher’s professional guidance, and your earnest support, your student will make gains in mathematics and have a positive experience with geometry.
Overview of Topics in Discovering Geometry

The arrangement of topics in Discovering Geometry: An Investigative Approach is carefully planned.

- Chapter 0 points out the geometry in the world around us, then gives students several opportunities to learn geometry while creating art. Teachers may assign all or part of this chapter, or skip it to save time.

- In Chapter 1, students learn some basic definitions, then practice writing good definitions of their own. They practice using a protractor to measure and draw angles.

- Chapter 2 is about using the most important tool of all—the mind. Students learn about both inductive reasoning (drawing conclusions based on noticing a pattern) and deductive reasoning (proving something is true by presenting facts and a logical argument).

- In Chapter 3, students learn to use the tools of geometry: compass, straightedge (a ruler without markings), and patty paper (squares of wax paper). As they use these tools to construct and compare geometric figures, they learn the properties of shapes. This first-hand knowledge is crucial for understanding concepts and for proving theorems—which they begin to do informally here.

- Chapters 4 through 6 are about discovering the properties of polygons and circles, then proving that these properties are true for an entire class of shapes. Students are using the reasoning techniques they learned in Chapter 2 and the construction techniques they learned in Chapter 3, and the lessons help them to build their knowledge systematically.

- Chapter 7 is another chapter that explores the connection between geometry and art. Students learn about transformations of figures—reflections, rotations, and translations—and how these relate to tiling patterns.

- Chapters 8 through 10 are about area, volume, and the Pythagorean Theorem. Here students focus on calculations and problem solving, often in the context of real-world problems.

- Chapter 11 is about similarity, or the proportionality of scaled figures.

- Chapter 12 is about trigonometry, or the study of measurements within triangles. Most high schools teach trigonometry in greater detail as part of a precalculus course, but this chapter serves as an introduction. The exercises continue to focus on calculation, problem solving, and real-world applications.

- Chapter 13 is an optional culminating chapter. Now that students have the proper foundational understanding of geometry, they are introduced to the postulates of geometry and asked to write formal proofs of some of the important theorems.

Chapter Summaries

More complete chapter summaries are given for each chapter in this book. The chapter content is briefly summarized, and important new words are italicized. A summary problem is presented, along with question prompts that you can use to get your student thinking. The summary problem is a comprehensive problem that will give you and your student a lot to talk about. The question prompts are followed by sample answers. Review exercises and solutions are provided at the end of the material for each chapter.
CHAPTER 0

Content Summary

In Chapter 0, students make connections between geometric ideas and things they’ve seen before: common shapes (circles, hexagons, pentagons), mirror symmetry (or reflectional symmetry), and lines and angles. Students study mathematics through art, which may be a new and motivating context for them. This approach is designed to connect to and expand the intuitive understanding they already have about shapes as a whole. In addition to reviewing familiar mathematical ideas, Chapter 0 introduces rotational symmetry, compass and straightedge constructions, and tessellations.

Your student’s teacher may select lessons from Chapter 0 to review skills and ideas that this class needs to understand better. The teacher can also use Chapter 0 as a gentle introduction to Discovering Geometry’s methods for bringing about deep understanding through investigating, working in groups, and using geometry tools and perhaps software. Meanwhile, you can use this time to help establish patterns in the way you’ll work with your student.

Rotational Symmetry

Most people say that a figure is symmetric if it has mirror symmetry—one side is a reflection of the other. There are other kinds of symmetry as well. Chapter 0 introduces rotational symmetry. A figure has 2-fold rotational symmetry if it looks the same after you turn it through half of a full turn—that is, through 180 degrees. It has 3-fold rotational symmetry if it looks the same after being turned one-third of a full turn—that is, through 120 degrees. And so on. The figure on the right, for example, has 3-fold rotational symmetry. All figures look the same after you turn them 360 degrees, so a figure that only satisfies this requirement is not considered to have rotational symmetry.

Students will revisit reflectional and rotational symmetry in the context of transformations in Chapter 7.

Tessellations

If you can make a bunch of tiles all the same shape and use those tiles to cover a flat surface, with no gaps, then you have a tessellation, or tiling pattern. In Chapter 0, students see a few tessellations. In Chapter 7, they’ll study properties of shapes that can be used in tessellations, as well as tessellations made with more than one shape.

Constructions

Mathematicians in ancient Greece believed that the most perfect shapes were circles and straight lines, so they set out to see what they could do with a compass (to make circles) and a straightedge (to make lines). For a compass, they needed a rope with a stake tied to one end. The straightedge was unmarked, so it couldn’t be used for measuring. Nevertheless, they found that with these tools they could construct many shapes and angles. Whatever can be drawn with these tools is called a geometric construction.

Although most of us don’t carry the same idea of perfect shapes as the Greeks did 2500 years ago, studying geometric constructions is valuable for students. It gives them a feeling for shapes and relationships. This feeling is very useful in studying geometric concepts and in logical reasoning.
Summary Problem

You and your student might discuss this summary problem from Chapter 0. It’s a good problem to revisit several times while working through the chapter.

What ideas from this chapter do you see in Hot Blocks, the fine art pictured on page 24, also shown below?

Discuss these questions with your student from the point of view of a student to your student:

● What kinds of symmetry does the picture have?
● What kinds of symmetry does each block have?
● Could you construct one of the blocks with straightedge and compass?
● What did the artist do to make each block appear three-dimensional?
● Could you construct a different but similar drawing with straightedge and compass?
● Is your own drawing more or less elegant than this one?
● Is it okay to discuss elegance in math?

Some of these questions have several possible valid answers. It is not important that you know all the answers. Instead, as you talk about the answers, make sure your student gives a good explanation of why an answer is reasonable. For example, an answer of “yes” or “no” is not enough. Encourage your student to ask questions too.

Sample Answers

If shading is ignored, the art has 2-fold rotational symmetry, with a vertical line of symmetry through the middle column of blocks and a horizontal line of symmetry between the middle rows of blocks. Each block, if shading is ignored, has 3-fold rotational symmetry and three lines of reflection. The patterns of shading make it appear three-dimensional. The two hexagons that make up each block can be constructed: Starting with circles and using their radii to mark off six equal arcs on the circles, points can be determined to draw equilateral hexagons from which to make blocks.

Although elegance is a matter of opinion, it is acceptable to talk about elegance in any discipline. Here the drawing could be considered elegant because of its artistic value or because of the mathematics it displays.
Chapter 0 • Review Exercises

Name ___________________________ Period ________ Date _____________

1. (Lesson 0.1) Which of the designs below have reflectional symmetry? Draw the lines of symmetry. Which of the designs have rotational symmetry?

![Designs with lines of symmetry drawn](image)

2. (Lesson 0.2) Name the basic tools of geometry. What are these tools used for?

3. (Lesson 0.3) Use your compass to create a 6-petal daisy design. Color it so it has reflectional symmetry but no rotational symmetry.

4. (Lesson 0.4) Use your straightedge and a copy of your daisy design from Exercise 3 to construct a regular hexagon. Use this to create one block from the Amish quilt tumbling block design pictured on page 14.
1. The compass is used for constructing circles and marking off equal distances, and the straightedge is used for drawing straight lines.

2. The compass is used for constructing circles and marking off equal distances, and the straightedge is used for drawing straight lines.

3. Rotational and reflectional

4. Rotational

Reflectional

Rotational and reflectional
Introducing Geometry

Content Summary
Chapter 1 introduces the building blocks and vocabulary of geometry. Though it refers to many geometric shapes that students have seen before, it emphasizes what’s required to make a good definition. Many of the investigations engage students in the process of writing definitions by presenting visual examples of shapes that belong to a group and shapes that do not belong to the group, called non-examples. This process begins to move students from thinking of shapes as their whole appearance toward thinking about their parts, and thinking about classes of shapes—for instance, thinking about what all rectangles have in common and what makes them rectangles. This lays the groundwork for understanding the properties of shapes, a higher level of thinking that becomes increasingly important in later chapters.

Definitions
A good definition can usually be put into the form “A [term being defined] is a [general group] that [has some characteristic].” For example, take the definition of a triangle.

A triangle is a polygon that has three sides.

For this definition to be clear, we must know what the terms used in the definition mean. What is a polygon? What is a side of a polygon? All terms used in a good definition should already be previously defined. But this leads to a problem: Where should we begin? We must agree on some terms to be understood without being defined. They can then be used to define the first definitions, which in turn can be used in other definitions. In geometry, these undefined terms are point, line, and plane. From these terms, we can define other terms, which we can then use to define the terms triangle and angle in the original example above.

Points are collinear if they lie on the same line.

A line segment is two points (called endpoints) and all the points between them that are collinear with the two points.

A polygon is a closed figure in a plane, formed by connecting line segments endpoint to endpoint (called vertices) with each segment intersecting exactly two others.

A side of a polygon is a line segment connecting consecutive vertices of the polygon.

While these definitions may not always be in the exact form described at the beginning of this section, most are based on classifying the term within a general group, then differentiating it from the group according to some characteristics.

Angles
Besides defining angle, the book discusses measures of angles and has students write definitions of different kinds of angles (right, acute, obtuse), as well as special pairs of angles (complementary, supplementary, vertical, linear pair).

Polygons
The book defines polygon and has students write definitions of special kinds of polygons (equilateral, equiangular, regular). Students also define different kinds of
Chapter 1 • Introducing Geometry (continued)

triangles (right, acute, obtuse, scalene, equilateral, isosceles) and quadrilaterals (trapezoid, kite, parallelogram, rhombus, rectangle, square).

Circles
A circle is the set of all points in a plane at a given distance from a given point. Students review some parts of a circle (center, radius, arc) and define other parts (chord, diameter, tangent).

Summary Problem
What geometric shapes does each of these solids contain?

Questions you might ask in your role as student to your student:

- What are the solids themselves called?
- Are there parts of the solids that are hidden in these drawings?
- What do the hidden parts look like?
- What shapes are mentioned in this chapter?
- Which of those shapes are contained in the solids? Can you color them in?
- Can you change these solids to some that do contain those shapes?
- What geometric relationships are mentioned in the chapter?
- Which of those relationships do shapes in the solids have?
- What measurements are mentioned in the chapter?
- Could you make any of those measurements on these solids?

Sample Answers
Together, the shapes—a rectangular pyramid and a cylinder with a cylindrical hole through it—contain line segments, collinear and coplanar points (vertices), acute and right angles, triangles, a rectangle, and concentric circles. The bottom of the cylinder, the interior of the cylindrical hole, and the back triangle in the pyramid are hidden from view. Other terms in the chapter include ray; vertical angles; polygons of several types, including several quadrilaterals; many lines related to circles, including tangent; and other solids, including cylinder, cones, and spheres. It’s possible to make some of those polygons by taking a “section,” or slicing through the pyramid.

Relationships mentioned in the chapter include bisect, congruent, perpendicular, parallel, skew lines, and supplementary or complementary angles. In the figures, it appears that: The base of the pyramid has edges that are perpendicular to each other and opposite edges congruent to each other; the bases of the cylinder are parallel to each other; and if the triangles are equilateral or isosceles, they have congruent angles and congruent edges. The drawing may not show the true angles or side lengths, but otherwise you could find the measure of each angle with a protractor and the length of each side with a ruler.
Chapter 1 • Review Exercises

Name ____________________________  Period ______  Date ____________

(Lesson 1.1, 1.2) Identify the following:

1. Midpoint: ______  2. Segment: ______

7. (Lesson 1.2) Mark the figure with the following information:

\[ \angle A \cong \angle C \]
\[ AB \cong BC \]
\[ BD \perp DC \]

(Lessons 1.3, 1.5, 1.6, 1.7) Draw and carefully label each figure.

8. Supplementary angles \(\angle ABD\) and \(\angle DBC\) with \(m\angle ABD = 90^\circ\)

9. Isosceles right triangle  10. Circle \(O\) with diameter \(AB\) and tangent \(CD\)

11. Parallelogram \(ABCD\)

12. (Lesson 1.4) \(ABCDE \cong FGHIJ\). The perimeter of \(ABCDE\) is 36 cm. Find these distances.

   a. \(AB = \) ______  b. \(HI = \) ______
   c. \(FJ = \) ______

13. (Lesson 1.8) Find the missing lengths. Assume all edges are perpendicular to each other.

   a. \(x = \) ______  b. \(y = \) ______
   c. \(z = \) ______

14. (Lesson 1.9) Create a Venn Diagram to show the relationships among triangles, isosceles triangles, and right triangles.
1. \(E\) is the midpoint of \(AD\) because \(AE \equiv ED\).
2. There are several segments, for example \(AE\) or \(AD\).
3. \(EC\) is the angle bisector because \(\angle BEC \equiv \angle CED\).
4. There are several rays, for example \(BF\) or \(EC\).
5. \(JI\)
6. There are several angles, for example \(\angle BEC\) or \(\angle BED\).

11. 

12. Because the two pentagons are congruent, \(ED = JI = 4x - 3\). The marks on the pentagon show that \(BC = ED\). Therefore, \(BC = ED = 4x - 3\). The perimeter of \(ABCDE = (x + 2) + (4x - 3) + 7 + (4x - 3) + 2x = 36\) cm. Solving for \(x\), you get \(x = 3\) cm. Substitute 3 in for \(x\) and use the fact that corresponding sides are congruent to find these side lengths.
   a. \(AB = 5\) cm 
   b. \(HI = 7\) cm 
   c. \(FI = 6\) cm

13. \(x = 4; y = 4; z = 6\)

14. Although it is possible for a triangle to be both isosceles and right, it is also possible for it to be one without being the other, so they are represented by two overlapping circles.
Reasoning in Geometry

Content Summary

One major purpose of any geometry course is to improve the ability of students to reason logically. Chapter 2 focuses on two basic kinds of reasoning: inductive and deductive. Students use inductive reasoning to identify visual and numerical patterns and to make predictions based on these patterns. Then students are introduced to the use of deductive reasoning to explain why these patterns are true. Students explore the relationships between the measures of angles formed by intersecting and parallel lines, make conjectures about these relationships, and learn how to use logical arguments to explain why these conjectures are true.

Inductive Reasoning

Every time you freeze your water bottle, the water expands. So you learn quickly not to put too much water in the bottle, to avoid having the top pop off or the bottle break. Reasoning in which you draw conclusions from experience is inductive. Mathematicians use inductive reasoning to guess at what might be true. For example, if you add up positive odd numbers starting at 1, you get a pattern.

\[
\begin{align*}
1 + 3 &= 4 \\
1 + 3 + 5 &= 9 \\
1 + 3 + 5 + 7 &= 16
\end{align*}
\]

The sums seem to be square numbers: \(4 = 2 \cdot 2, 9 = 3 \cdot 3, 16 = 4 \cdot 4\). Even if you “add up” only the first odd number, 1, the sum is a square: \(1 \cdot 1\). You might conclude that the sum of any number of positive consecutive odd numbers, beginning with 1, is a square number.

But inductive reasoning is not foolproof. It may be that one time you’ll put the water bottle in the freezer and the water won’t expand. (Perhaps the freezer isn’t working, or the water is flavored in some way.) So mathematicians are not satisfied with inductive reasoning. Inductive reasoning leads only to guesses, or conjectures. A conjecture becomes a mathematical fact, or theorem, only if someone shows that it’s the conclusion of deductive reasoning.

Deductive Reasoning

Deductive reasoning, also called proof, is reasoning from proven facts using logically valid steps to arrive at a conclusion. A proof can serve many purposes. Mathematicians often use proof to verify that a conjecture is true for all cases, not just for those they examined, or to convince others. Proofs often help answer the question: Why? The use of proof to explain why is a natural extension for students at this point in the course and helps to deepen their understanding. This chapter stresses this illumination purpose of proof.

If you took a geometry course, you may have encountered proofs displayed in two columns: a column of statements and a column of reasons, with each statement justified by a reason. However, most students are overwhelmed by this approach. They find the format difficult to follow and miss the big picture. In Discovering Geometry, two-column proofs come in Chapter 13 with the study of geometry as a mathematical system, at which point students are at the appropriate developmental
Chapter 2 • Reasoning in Geometry (continued)

level. For now, students are encouraged to use informal deductive arguments written in a paragraph form. In Chapter 4, they will be introduced to other formats for presenting proofs.

Reasoning Strategies
The most difficult part of the process for writing a deductive argument is to determine the underlying logic of the argument and what information to include. Starting in Chapter 2 and continuing throughout the book, students are taught reasoning strategies, ways of thinking that help to construct a deductive argument. You may wish to discuss these ways of thinking with your student. The first three of these reasoning strategies are presented in this chapter, and one more strategy is introduced in each subsequent chapter as indicated.

● Draw a labeled diagram and mark what you know.
● Represent a situation algebraically.
● Apply previous conjectures and definitions.
● Break a problem into parts (Chapter 3)
● Add an auxiliary line (Chapter 4)
● Think backward (Chapter 5)

Summary Problem
Draw two intersecting lines. What do you notice about the vertical angles? Can you explain any patterns you see?

Questions you might ask in your role as student to your student:

● What does the term vertical angles mean?
● Which pairs of angles are vertical angles?
● What conjecture might you state about the measures of vertical angles?
● Does the conjecture you’re making apply to all pairs of intersecting lines that you’ve tried, or can you find a counterexample?
● Do you think your conjecture holds for all pairs of intersecting lines?
● How would you show that it’s true for all pairs?

Sample Answers
When two lines intersect, they form four distinct angles. Two nonadjacent angles formed by the intersecting lines are vertical angles. Two intersecting lines form two pairs of vertical angles. The angles in each pair have equal measures. Explaining why could involve talking about straight lines and the various pairs of adjacent angles, or about a rotation around the vertex. You can write a deductive argument to show that the Vertical Angles Conjecture follows logically from the Linear Pair Conjecture, as shown on page 124.
Chapter 2 • Review Exercises

1. (Lesson 2.1) Use the rule provided to generate the next five terms of the sequence.
   \[2, 5, 9, 14, \ldots, \frac{n(n + 3)}{2}, \ldots\]

2. (Lessons 2.2, 2.3, 2.4) Find the rule for the number of circles in the \(n^{th}\) figure, and use that to find the number of circles in the 20th figure. Are you using inductive or deductive reasoning to answer this question?

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>(n)</th>
<th>(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of circles</td>
<td>1</td>
<td>6</td>
<td>11</td>
<td>16</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
</tbody>
</table>

3. (Lessons 2.5, 2.6) Find the measure of each lettered angle. Is line \(l\) parallel to \(m\)? Explain your reasoning.

4. (Lesson 2.6) Lines \(l\) and \(m\) are parallel to each other. Find \(x\).
SOLUTIONS TO CHAPTER 2 REVIEW EXERCISES

1. 20, 27, 35, 44, 54

The first four terms come from substituting 1, 2, 3, and 4 for n in the given rule. Substitute 5, 6, 7, 8, and 9 for n to find the next five terms. See bottom of the page.

2. $5n - 4$ (or $5(n - 1) + 1$, which is equivalent); 96; inductive reasoning
   First, look for the difference between the terms. In this case we add five to each term to get the next term.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of circles</td>
<td>1</td>
<td>6</td>
<td>11</td>
<td>16</td>
<td>…</td>
</tr>
</tbody>
</table>

Because the difference between consecutive terms is always five, the rule is $5n + "something." Let c stand for the unknown "something," so the rule is $5n + c$. To find $c$, replace $n$ in the rule with one term number. For example, try $n = 3$ and set the expression equal to 11, the number of circles in the 3rd figure.

$5(3) + c = 11$
$15 + c = 11$
$c = -4$

Therefore, the rule is $5n - 4$. To find out how many circles there are in the 20th figure, substitute 20 for $n$ in the rule.

$5(20) - 4 = 96$

3. $a = 55°$ by the Vertical Angles Conjecture.
   $b = 126°$ by the Vertical Angles Conjecture.
   $c = 54°$ by the Linear Pair Conjecture.
   If lines $l$ and $m$ were parallel, then $c$ would equal 55° by the Corresponding Angles Conjecture. However, $c = 54°$ as seen above, so $l$ and $m$ are not parallel.

4. $40 + 3x + 4x = 180$ Corresponding Angles Conjecture and Linear Pair Conjecture.
   
   $40 + 7x = 180$ Combine like terms.
   $7x = 140$ Subtract 40 from both sides.
   $x = 20$ Divide both sides by 7.

Figure number

<table>
<thead>
<tr>
<th>n</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>n(n + 3)</td>
<td>5(5 + 3)</td>
<td>6(6 + 3)</td>
<td>7(7 + 3)</td>
<td>8(8 + 3)</td>
<td>9(9 + 3)</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>27</td>
<td>35</td>
<td>44</td>
<td>54</td>
</tr>
</tbody>
</table>
CHAPTER 3

Using Tools of Geometry

At this point in the course, you might think about how you and your student are interacting. For example, are you being a student to your student? Are you asking questions and letting your student explain? Do you explain little enough so that your student is becoming an independent learner and thinker? Do you answer questions that your student hasn’t asked? Telling your student too much can waste time, because he or she might not understand; this can lead to your student feeling overwhelmed. Deeper understanding can result when you allow your student to teach the concept or skill to you and others.

Content Summary

In Chapter 3, students use hands-on constructions to develop an intuitive sense of the properties of shapes. This allows for a different mode of understanding how parts of a figure are related to the whole. Students learn to duplicate segments and angles, then work with perpendicular and parallel lines, angle and segment bisectors, and concurrent lines. An underlying concept is determination: What properties determine a figure’s shape? That is, what properties are needed so that those properties determine one and only one shape? The idea of determination is addressed through geometric constructions.

Geometric Constructions

Students learn how to duplicate segments and angles, bisect segments and angles, and construct perpendicular and parallel lines. In addition to learning the classical compass and straightedge constructions, students learn how to use patty paper, small squares of waxed paper usually used between burger patties, as a unique geometry tool for constructions. You can think of geometric constructions as a game: Try to draw a figure, such as a square, using only a straightedge and compass and no measurements. The solutions can be applied to real-life problems, such as laying out the foundation for a building, but modern technology offers simpler methods. Try to help your student enjoy the game. For over 2500 years, this game has been giving students a hands-on understanding of properties that determine the shape of a figure. An understanding of determination will be especially helpful in the study of triangle congruence in Chapter 4.

(continued)
Chapter 3 • Using Tools of Geometry (continued)

Summary Problem
Draw three line segments on a sheet of paper. Using only the construction tools (unmarked straightedge and compass), duplicate them to make a triangle, if possible. If you succeed, make a few triangles. Then construct whatever you know how to construct on the triangle’s sides and angles. What patterns do you see? What conjectures can you make? Can you give reasons those conjectures might be true?

Questions you might ask in your role as student to your student:

- Is it really possible to make a triangle from copies of those line segments without doing any measuring?
- Is it ever impossible to make a triangle from three given segments?
- What happens when you construct several triangles using the same three segments? Do three segments determine a triangle?
- What happens if you construct the perpendicular bisector of each side of the triangle?
- What happens if you construct the bisector of each angle of the triangle?
- What happens if you construct the midpoint of each side of the triangle and join each midpoint to other points?
- What else can you construct on this triangle, and what other patterns do you see?

Sample Answers for the Chapter 3 Summary Problem
As long as the sum of any two lines is longer than the third, a triangle can be constructed. Constructing several triangles from the same three segments always yields the same triangle (although it may be a mirror image). So, provided that a triangle can be constructed, the triangle is determined. Once you have three segments that can be duplicated to form a triangle, your student can use the copies of the triangle as they explore. The perpendicular bisector of each side can be constructed on one copy of the triangle and the angle bisectors on another. Students will find points of concurrency, points at which three lines intersect. Students will also find a relationship between the two parts of a segment determined by the point of concurrency for the medians, and relationships among some of these points of concurrency if they do the Euler Line Exploration at the end of the chapter. Your student may explore other relationships.
Chapter 3 • Review Exercises

1. (Lesson 3.1, 3.2) Duplicate $AB$ and find its perpendicular bisector.

\[ \begin{array}{c}
A \\
\hline \hline \hline
B
\end{array} \]

2. (Lesson 3.2, 3.3) Identify the parts of $\triangle ABC$:
   a. Median: ______
   b. Altitude: ______
   c. Midsegment: ______

\[ \begin{array}{c}
A \\
\hline \hline \hline
F \\
\hline \hline \hline
G \\
\hline \hline \hline
B
\end{array} \]

3. (Lesson 3.1, 3.4) Duplicate $\triangle DEF$ and construct its angle bisector.

\[ \begin{array}{c}
D \\
\hline \hline \hline
C \\
\hline \hline \hline
E \\
\hline \hline \hline
F
\end{array} \]

4. (Lesson 3.5, 3.6) Construct an isosceles trapezoid.

5. (Lesson 3.7) Name each center.
   a. 
   b. 
   c. 
   d. 

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SOLUTIONS TO CHAPTER 3 REVIEW EXERCISES

1. Duplication:

```
A
  |
  |
  |
  |
C
```

Step 1

```
A
  |
  |
  |
  |
B
```

Step 2

```
D
  |
  |
  |
  |
C
```

Step 3

Perpendicular Bisector

2. Median = $\overline{AE}$ because it connects the vertex to the midpoint

Altitude = $\overline{AD}$ because it is the perpendicular segment from a vertex to the line containing the opposite side.

Midsegment = $\overline{FG}$ because it connects the midpoints of two sides.

3. Duplication:

```
E
  |
  |
  |
  |
F
```

Step 1

```
G
  |
  |
  |
  |
```

Step 2

```
E
  |
  |
  |
  |
F
```

Step 3

```
G
  |
  |
  |
  |
```

Step 4

```
G
  |
  |
  |
  |
```

Step 5

Angle bisector:

4.

```
A
  |
  |
  |
  |
B
```

```
C
  |
  |
  |
  |
```

```
D
  |
  |
  |
  |
```

```
E
  |
  |
  |
  |
```

```
F
  |
  |
  |
  |
```

5. a. Orthocenter (intersection of altitudes)

b. Centroid (intersection of medians)

c. Circumcenter (intersection of perpendicular bisectors)

d. Incenter (intersection of angle bisectors)
**CHAPTER 4**

**Discovering and Proving Triangle Properties**

**Content Summary**

In Chapter 4, students explore properties of triangles and the conditions that guarantee that two triangles are congruent. At first students make conjectures about the sum of interior and exterior angles, properties of isosceles triangles, and inequality relationships among sides and angles of triangles. Then they explore the characteristics needed to determine the congruence of two triangles and, finally, learn how to use this to prove their conjectures.

**Angle Relationships in Triangles**

Students experiment, look for patterns, and make conjectures about parts of triangles. These key conjectures result from their investigations:

- The sum of the angles of any triangle is $180^\circ$.
- Two angles of a triangle are congruent if and only if two sides of the triangle are congruent.
- The measure of an exterior angle of a triangle equals the sum of measures of the two interior angles that are not adjacent to that exterior angle.

**Triangle Congruence**

The idea of congruence serves as a bridge between properties of a single triangle and properties shared by two or more triangles. In one sense, congruence is about determination. Knowing all three angles and all three sides certainly determines a triangle. In other words, if you draw a second triangle with all sides and angles congruent to those in the first triangle, the second triangle will be congruent to the first. Essentially, it will be the same triangle. So, knowing three angles and three sides guarantees the size and shape of the triangle, and all triangles that share that set of measurements are guaranteed to be congruent to each other. But is a triangle determined by fewer than six pieces of information? For example, is knowing three angles enough to determine a triangle? Is knowing two sides and an angle sufficient? Likewise, how can you tell if two triangles are congruent? Are they congruent if their three angles have the same measure? Or, if two sides and an angle are the same? The book calls these conjectures *congruence shortcuts*. Those shortcuts that are sufficient to guarantee congruence are listed at right.

- **Side-Side-Side (SSS)**
  - Three pairs of congruent sides

- **Side-Angle-Side (SAS)**
  - Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

- **Angle-Side-Angle (ASA)**
  - Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

- **Side-Angle-Angle (SAA)**
  - Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

(continued)
Chapter 4 • Discovering and Proving Triangle Properties (continued)

Proof
An important reason for developing congruence shortcuts for triangles is to prove other properties of geometric figures. Chapter 4 introduces two formats for presenting proofs that are used throughout the remainder of the course. The paragraph proof, introduced at the beginning of the chapter, is a deductive argument that uses written sentences to support its claims with reasons. The flowchart proof, introduced near the end of the chapter, places statements in boxes connected by arrows to show the flow of logic, with the logical reasons presented below each box. In the last three lessons of the chapter, students apply triangle congruence shortcuts using these proof formats to prove triangle properties they discovered throughout the chapter.

Summary Problem
Suppose you know the length of the altitude to the base of an isosceles triangle and the measure of an angle between the base and another side. Is this enough information to determine a triangle, or are different triangles possible?

Questions you might ask in your role as student to your student:

● Does it help to draw particular altitudes and angles and try to make more than one triangle with the given properties?
● Do you think more than one triangle is possible? Why?
● Can you use the Triangle Sum Conjecture to help explain why?
● Can you use the Isosceles Triangle Conjecture to help explain why?
● Can you use congruence shortcuts to help explain why?
● Can you use the Vertex Angle Bisector Conjecture to help explain why?
● What if the triangle isn’t isosceles?

Sample Answers
Making and labeling a diagram is a good technique to help you think about a problem. In this case, your drawing will show you that there is only one possible triangle with an altitude of the specific length you drew and the angle you drew between the triangle’s base and another side. To explain why, your student will use several of the conjectures. Here is one explanation, but encourage your student to give other explanations.

Because you know one of the base angles, you also know the other by the Isosceles Triangle Conjecture. The altitude of the isosceles triangle splits it into two right triangles because an altitude is defined to be perpendicular to the base. Both right triangles have the same two angles and one side (actually, two sides if you consider the shared altitude). By the Side-Angle-Angle Congruence Conjecture, all such triangles are congruent. Thus, if you construct a new isosceles triangle with the same given altitude and base angle, it will be composed of two of the same congruent right triangles, so it is determined.

Other explanations might use the Vertex Angle Bisector Conjecture along with any of the triangle congruence shortcuts to help explain why the two halves of the isosceles triangle are congruent.

Note that when these arguments are applied to a triangle that is not isosceles, they fail. The second “base angle” of the triangle need not be congruent to the first. To see this, encourage your student to draw a few noncongruent triangles that have a given altitude and a given angle between the base and one of the legs. In the diagram at right, if you are given \( \angle A \), \( AB \), and the altitude \( AD \), you can locate point \( C \) anywhere along \( AD \) if the triangle is not isosceles.
Chapter 4 • Review Exercises

(Lesson 4.1, 4.2) For Exercises 1 and 2, find the missing measures.

1. Calculate the measure of each lettered angle and explain how you found it.

2. The perimeter of \( \triangle ABC \) is 36 in.
   \[
   BC = \ ? \\
   AB = \ ? \\
   m\angle C = \ ?
   \]

(Lesson 4.3) For Exercises 3 and 4, arrange the three unknown measures in order from greatest to least.

3. 

(Lesson 4.4, 4.5) For Exercises 5 and 6, decide whether the triangles are congruent. If they are, name the congruence shortcut you used.

5. 

6. 

7. (Lesson 4.6, 4.7) Create a flowchart proof to show that \( AB \cong CB \).

8. (Lesson 4.8) Write a paragraph proof to show that \( AB \cong CB \).
Solutions to Chapter 4 Review Exercises

1. \( c = 70^\circ \) Supplement of 110°.
   \( a = b \) Isosceles Triangle Conjecture.
   \( a + b + 70^\circ = 180^\circ \) Triangle Sum Conjecture.
   \( a + a + 70^\circ = 180^\circ \) Substitution.
   \( 2a + 70^\circ = 180^\circ \) Combine like terms.
   \( 2a = 110^\circ \) Subtraction.
   \( a = 55^\circ \) Division.
   \( b = 55^\circ \) Substitution.

2. \( BC = 15 \) in. Definition of an isosceles triangle.
   \( AB + 15 + 15 = 36 \) Perimeter.
   \( 30 + AB = 36 \) Addition.
   \( AB = 6 \) in. Subtraction.
   \( m\angle B = 75^\circ \) Isosceles Triangle Conjecture.
   \( 75^\circ + 75^\circ + m\angle C = 180^\circ \) Triangle Sum Conjecture.
   \( 150^\circ + m\angle C = 180^\circ \) Addition.
   \( m\angle C = 30^\circ \) Subtraction.

3. \( c > b > a \) \( c \) is opposite the largest angle and \( a \) is opposite the smallest angle.

4. \( f > d > e \) \( f \) is opposite the largest side and \( e \) is opposite the smallest side.

5. Yes, \( \triangle ABC \cong \triangle CDA \) by SAS.

6. Yes, the triangles are congruent by SSS.

7. See bottom of the page.

8. We are given that \( \overline{AD} \cong \overline{CD} \) and \( \angle ADB \cong \angle CDB \).
   \( \overline{BD} \cong \overline{BD} \) because it is the same segment,
   so \( \triangle ABD \cong \triangle CBD \) by SAS. Therefore, \( \overline{AB} \cong \overline{CB} \)
   by CPCTC (Corresponding Parts of Congruent Triangles are Congruent).
CHAPTER 5

Discovering and Proving Polygon Properties

Content Summary
Chapter 5 extends the explorations of triangle properties from the previous chapter to examine properties shared by all polygons. Students begin by investigating the sums of interior and exterior angles of any polygon. The chapter then focuses on quadrilaterals, which are polygons with four sides. Students explore relationships among the sides, angles, and diagonals of different special quadrilaterals, including the parallelogram family.

Polygons
The chapter begins with conjectures about polygons in general. Students experiment to form conjectures about the sum of the angles of any polygon, and the sum of the exterior angles of any polygon. They write a paragraph proof of the first conjecture, relying on the Triangle Sum Conjecture from Chapter 4.

Quadrilaterals
The book considers properties of three categories of quadrilaterals, as in the diagram: kites, trapezoids, and parallelograms.

Students explore two kinds of parallelograms, rhombuses and rectangles, as well as squares, which are both rhombuses and rectangles. Students discover properties of all types of quadrilaterals, including how their diagonals are related. In the case of trapezoids, students investigate midsegments, which they relate to midsegments of triangles.

Properties of various quadrilaterals can be seen from their symmetry. A kite has reflectional symmetry across the diagonal through its vertex angles; an isosceles trapezoid has reflectional symmetry across the line through the midpoints of the parallel sides; and a parallelogram has 2-fold rotational symmetry about the point at which its diagonals intersect. These symmetries can help explain why certain pairs of segments or angles are congruent or perpendicular.

Summary Problem
Make a copy of the quadrilaterals diagram above, but with large boxes. Write in each box the properties of that kind of figure as you encounter them in the book.

Questions you might ask in your role as student to your student:

- If you add a box to the top of the diagram for polygons in general, what properties can you put into that box?
- What other kinds of polygons might go into an expanded diagram?
- Where might isosceles trapezoids be added to your diagram?
- Where might darts be added to your diagram?
- What properties can you think of that aren’t already in your diagram?
- Do you see any patterns in what properties are shared by various kinds of polygons?
Chapter 5 • Discovering and Proving Polygon Properties (continued)

Sample Answers

**Polygon**
The sum of the exterior angles is 360°. A polygon of $n$ sides has $\frac{n(n - 3)}{2}$ diagonals and the sum of its interior angles is $180° (n - 2)$.

**Quadrilateral**
A quadrilateral is a polygon with four sides. The sum of the interior angles is 360°.

**Kite**
A kite has exactly two distinct pairs of congruent sides. The nonvertex angles are congruent. The diagonals are perpendicular. The diagonal between the vertex angles bisects the other diagonal. It has exactly one line of symmetry.

**Trapezoid**
A trapezoid has exactly one pair of parallel sides. Two pairs of adjacent angles are supplementary.

**Isosceles trapezoid**
An isosceles trapezoid has two pairs of congruent angles and at least two congruent sides. Its diagonals are congruent. It has one line of symmetry.

**Parallelogram**
The opposite sides of a parallelogram are congruent and parallel. The opposite angles are congruent. Adjacent angles are supplementary. The diagonals bisect each other.

**Rhombus**
A rhombus has all the characteristics of a parallelogram. Four sides are congruent. Diagonals are perpendicular. It has two lines of symmetry and 2-fold rotational symmetry.

**Rectangle**
A rectangle has all the characteristics of a parallelogram. Angles are all right angles. It has two lines of symmetry and 2-fold rotational symmetry.

**Square**
A square has all the characteristics of a rhombus and of a rectangle. It has four lines of symmetry and 4-fold rotational symmetry.

Making each box in the shape of the polygon whose properties it contains might make the chart more interesting.
Chapter 5 • Review Exercises

Name ___________________________ Period _________ Date ______________

1. (Lessons 5.1, 5.2) Find the sum of the measures of the interior angles of a regular 14-gon. Then find the sum of the exterior angles.

(Lessons 5.1, 5.2, 5.4) For Exercises 2 and 3, find the lettered measures in each figure.

2. 

3. Given that $\overline{CD} \parallel \overline{AF}$,

$BE = \ldots$.

$m \angle ABE = \ldots$.

$m \angle CDF = \ldots$.

4. (Lesson 5.3) Given kite $ABCD$, find the missing measures.

5. (Lesson 5.5) The perimeter of parallelogram $ABCD$ is 46 in. Find the lengths of the sides.

6. (Lesson 5.6, 5.7) Draw a diagram and write a paragraph proof to show that the diagonals of a rectangle are congruent.
1. Interior Angles:
   Interior angle sum = 180 (n - 2) = 180 (14 - 2) = 2160°
   Exterior Angles = 360° for all polygons

2. For the hexagon:
   Interior angle sum = 180 (n - 2) = 180 (6 - 2) = 720°
   Each angle = \( \frac{720°}{6} = 120° \)
   \( a = 120° \)
   \( b = 60° \) Linear pair.
   \( c = 60° \) Triangle sum.

3. \( BE = \frac{15 + 32}{2} = 23.5 \) cm Midsegment.
   \( m\angle ABE = 110° \) Linear pair.
   \( m\angle CDF = 105° \) Supplementary angles.

4. \( a = 90° \) Diagonals of a kite are perpendicular.
   \( b = 10 \) cm Definition of a kite.
   \( c = 30° \) Triangle sum.

5. \( 2(3x - 1) + 2(x + 4) = 46 \) Opposite sides of a parallelogram are congruent.
   \( 6x - 2 + 2x + 8 = 46 \) Distributive property.
   \( 8x + 6 = 46 \) Combine like terms.
   \( 8x = 40 \) Subtraction.
   \( x = 5 \) Division.

   \( AB = 3(5) - 1 = 14 \) in. Substitution.
   \( AD = 5 + 4 = 9 \) in. Substitution.

6. Sample answer:

   By definition, all angles of a rectangle are congruent, so \( \angle ABC \cong \angle DCB \). A rectangle, like any parallelogram, has opposite sides congruent, so \( \overline{AB} \cong \overline{DC} \). Because it is the same segment, \( \overline{BC} \cong \overline{BC} \). Thus \( \triangle ABC \cong \triangle DCB \) by SAS, and \( \overline{AC} \cong \overline{DB} \) by CPCTC. Therefore, the diagonals of a rectangle are congruent.
CHAPTER 6

Discovering and Proving Circle Properties

Content Summary
In Chapter 6, students continue to build their understanding of geometry as they explore properties of circles. Some of these properties are associated with line segments related to circles; other properties are associated with arcs and angles. A circle is defined as a set of points equidistant from a fixed point, its center.

Line Segments Related to Circles
The best-known line segments related to a circle are its radius and diameter. Actually, the word radius can refer either to a line segment between a point on the circle and the center, or to the length of such a line segment. Similarly, diameter means either a line segment that has endpoints on the circle and passes through the center, or the length of such a line segment.

The diameter is a special case because it is the longest chord of a circle; a chord is a line segment whose endpoints are on the circle. Another line segment associated with circles is a tangent segment, which touches the circle at just one point and lies on a tangent line, which also touches the circle at just one point and is perpendicular to the radius at this point. Students learned about these segments in Chapter 1, and Lesson 6.1 provides a quick review.

Arcs and Angles
A piece of the circle itself is an arc. If you join each endpoint of an arc to the center of the circle, you form the central angle that intercepts the arc. The size of the arc can be expressed in degrees—the number of degrees in the arc's central angle. This chapter explores several such relationships among arcs, angles, and segments. Students also write paragraph and flowchart proofs to confirm the universality of these relationships.

The size of the arc can also be expressed in length. The arc length is calculated by using the total circumference, or the distance around the circle. The number π is defined to be the circumference of any circle divided by that circle's diameter; or, the circumference is π times the diameter. For example, if an arc is 1/4 of the complete circle, then its central angle measures 1/4 of 360°, and its length is 1/4 of the circle's circumference.

Summary Problem
Draw a diagram of a central angle intercepting a chord of a circle and its arc, as shown in the picture.

Move points A, B, and C to different locations to illustrate the ideas of the chapter.

Questions you might ask in your role as student to your student:

● What concepts are illustrated in the original drawing?
● How could you move each of the points A, B, and C to show an inscribed angle?
● How could you move each of the points A, B, and C to show tangent segments?

(continued)
Chapter 6 • Discovering and Proving Circle Properties (continued)

- How could you move points A, B, and C to show an angle inscribed in a semicircle?
- How could you move points A, B, and C to show parallel lines intercepting congruent arcs?

Sample Answers
The original drawing shows a central angle, a sector, a chord, and an arc. If the center, C, is moved to lie on the circle, an inscribed angle is formed.

If C is moved to be outside the circle, and A and B are moved to make \( \overline{AC} \) and \( \overline{BC} \) tangents, then those segments are congruent. Or, the chord in the original diagram could be rotated at one of its ends until it becomes a tangent segment.

If C is moved until \( \overline{AC} \) is a diameter and B remains on the circle, \( \angle ABC \) is a right angle inscribed in a circle.

You would need to add another point and put C on the circle to show two chords. Parallel chords intercept equal arcs if they are equidistant from the center.

Of course many other answers are possible. Encourage your student to think of multiple ways that A, B, and C could be moved to illustrate these same concepts.
Chapter 6 • Review Exercises

Name ___________________________ Period _________ Date ___________

1. (Lesson 6.1) Given tangent $\overline{AB}$, find $m\angle OAB$, $m\angle AOB$, and $m\angle ABO$.

2. (Lessons 6.2, 6.3) Find the unknown measures or lengths.

3. (Lesson 6.3) $\triangle ABC$ is an equilateral triangle. Find $m\overline{AB}$.

4. (Lesson 6.4) Write a paragraph proof to prove the following:
   Given: Circle $A$ with diameters $\overline{EC}$ and $\overline{BD}$.
   Prove: $\overline{ED} \cong \overline{BC}$

5. (Lessons 6.5, 6.7) Given that the circumference of circle $A$ is $24\pi$ in., find the radius of the circle and the length of $\overline{BDC}$.

6. (Lessons 6.1, 6.3) $\overline{AB}$ and $\overline{BC}$ are tangents to the circle as shown. $\overline{AC} \parallel \overline{ED}$. Find $a$ and $b$. 
4. Diameters $\overline{CE}$ and $\overline{BD}$ on circle $A$ intersect to form congruent vertical angles, so $m\angle BAC = m\angle DAE$. The measure of an arc equals the measure of its central angle. Therefore, $m\overarc{BC} = m\overarc{ED}$ because the measures of their central angles are equal.

5. $C = 2\pi r = 24\pi$ in.; therefore, $r = 12$ in.
   
   $m\overarc{BDC} = 360\degree - 120\degree = 240\degree$
   
   length of $\overarc{BDC} = \frac{240}{360} (24\pi) = 16\pi$ in.

6. $m\overarc{CD} = m\overarc{AE} = a$

   Parallel secants intercept congruent arcs.

   $100\degree + a + a + 140\degree = 360\degree$
   
   $a = 60\degree$
   
   Solve.

   $b = 15$ $m$

   Tangent segments from the same point are congruent.
CHAPTER 7

Transformations and Tessellations

Content Summary
Thinking about ideas from different perspectives can lead to deeper understanding. For example, geometric transformations can help students deepen their understanding of congruence and symmetry.

You can think of a geometric transformation as a regular change of a figure in the plane. For example, a figure may be shifted to the right 5. Or, a figure may be enlarged to twice its original size.

Chapter 7 focuses on transformations that don’t change the size or shape of figures. These transformations are called isometries. Expansions and contractions, called dilations, are studied in Chapter 11.

Isometries
There are three major kinds of isometries: translations, reflections, and rotations.

Translations are simply shifts. Students use translations when discussing tessellations, where a single shape is translated (shifted) repeatedly in different directions to cover the plane without any gaps or overlaps.

Reflections flip a shape across a line to make a mirror image. If there’s a line through which a shape can be reflected to lay the image exactly on top of the original, then the figure has reflectional symmetry, as students saw in Chapter 0. Reflections can be used in designing figures that will tessellate the plane. They can also be used to help find the shortest path from one object to a line and then to another object.

Rotations rotate an object around a point. If there’s a point around which a shape can be rotated through some angle (less than 360°) to get back to exactly the same shape, then the figure has rotational symmetry (also previewed in Chapter 0). Rotations can also be used in designing tessellations.

Isometries give students a new way of thinking about congruence. Two figures are congruent if one can be transformed into the other using an isometry.

Compositions of Isometries
One transformation followed by another is the composition of those transformations. This chapter considers compositions of two reflections: reflections across parallel lines (resulting in a translation) and reflections across nonparallel lines (resulting in a rotation). In the context of tessellations, this chapter also examines glide reflections, which are compositions of a translation and a reflection.

(continued)
Chapter 7 • Transformations and Tessellations (continued)

Summary Problem

Have your student trace this tessellation onto tracing paper or wax paper. Ask how it illustrates the concepts under consideration.

Questions you might ask in your role as student to your student:

- What isometries might be used to change one part of the figure to another?
- What is the underlying grid of this tessellation?
- Could this tessellation be made with just translations?
- What kinds of symmetry does the entire figure have?

Sample Answers

The tessellation, based on a four-by-four grid of quadrilaterals, illustrates many transformations. Each of the dark figures can be translated to fit onto a dark figure two rows or columns away. Or, it can be rotated 180° to fit onto a light figure in the same row. To fit onto a light figure in an adjacent row, the dark figure must be reflected vertically, then translated. (All three transformations just described are also true for each of the light figures.) Any block of four figures that meet at one point can be translated to fill the entire plane.
Chapter 7 • Review Exercises

Name ___________________________ Period ___________ Date ___________

1. (Lesson 7.1) Reflect the figure across $\overline{PQ}$ and rotate it 180° around point $M$. Does your answer change if you rotate it first and then reflect it?

2. (Lesson 7.2) Translate $\triangle ABC$ using the rule $(x, y) \rightarrow (x - 3, y + 1)$.

3. (Lesson 7.3) Name the single rotation that can replace the composition of these two rotations around the same center of rotation: 36° and 120°.

4. (Lesson 7.1) What capital letters have horizontal but not vertical symmetry?

5. (Lesson 7.1) What capital letters have horizontal, vertical, and rotational symmetry?
1. Reflection:

Rotation:

No, the final answer is the same.

2.

3. $36^\circ + 120^\circ = 156^\circ$

4. B, C, D, E, K

5. H, I, O, X
Content Summary

Your student may have already seen formulas for the areas of standard geometric figures. If so, ask for the meaning of the formulas. Some appropriate questions might be, “What does area mean?” and “Does this formula still apply if the figure is rotated to have a different base?” Focus on the fact that area is measured by the number of square units that could fill a region.

Areas of Polygons

Some students lack an important concept regarding area: When a figure is cut up and its pieces are rearranged, the resulting figure has the same area as the original. With this concept, students can understand why the area formulas for parallelograms, triangles, and trapezoids can be found using the formula for the area of a rectangle, and how the areas of other plane figures can be found using areas of triangles.

Areas of Circles and Parts of Circles

Mathematicians in ancient times realized that the ratio of circumference to diameter is the same for every circle, a number we now call \( \pi \). What wasn’t realized until later is that this same number appears in a formula for the area of the region enclosed by a circle. Discovering Geometry shows ways of connecting the ideas of circumference and area to make the formulas more understandable. Students also learn how to find the areas of the parts of a circle shown below by relating these shapes to circles and triangles.

Surface Area

The surface area of a solid refers to the area of the surface, which is actually two-dimensional. A solid’s surface is often composed of standard shapes; to find the surface, your students will find the sum of the areas of these shapes.

Some solids have a base or two whose areas must be included in the sum. Also included are the areas of the sides (or lateral faces), which in this book are primarily rectangles or triangles. The lateral face of a cylinder, when flattened out, is a rectangle. The lateral face of a cone, when flattened out, is a sector of a circle. The surface area of a sphere is considered in Chapter 10.
Chapter 8 • Area (continued)

Summary Problem
What is the surface area of this stone sculpture? Each of the parallel bases began as a circle. Half of each circle was left untouched. The other half was cut three times, leaving three sides of a regular hexagon.

Questions you might ask in your role as student to your student:

- Can you find the areas of parts of the figure?
- How does a regular hexagon relate to the circle in which it’s inscribed?
- How does the radius of the circle relate to the length of one of the straight sides?
- For an extra challenge, consider what would happen if more stone were cut from the sculpture until it tapered evenly from the lower base to a point at the center of where the top base is now. What would the surface area be?

Sample Answers
Calculating surface area involves identifying all the surfaces, then systematically calculating and totaling their areas. Encourage your student to be systematic and organized. When doing the calculations, students should check their work. It also helps to estimate the answer first to be sure no large errors were made.

Each base consists of a semicircle and half of a regular hexagon. As shown in diagram at right, the half-hexagon can be divided into three congruent equilateral triangles, so the radius of the semicircle is 8 in. The area of the semicircle is $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(8)^2 = 32\pi \text{ in}^2$. The area of each equilateral triangle is $\frac{1}{2}(8)(4\sqrt{3}) = 16\sqrt{3} \text{ in}^2$, so the area of the half-hexagon is $48\sqrt{3} \text{ in}^2$. The area of each base, then, is $48\sqrt{3} + 32\pi \text{ in}^2$.

Each of the three rectangular lateral faces has area $8 \cdot 18 = 144 \text{ in}^2$. If you were to cut off the curved lateral face and lay it flat, it would form a rectangle of height 18 in. The width of that rectangle would be half the circle’s circumference, or $\frac{1}{2} \cdot 2\pi r = \frac{1}{2} \cdot 2\pi(8) = 8\pi \text{ in}$. The area of that lateral face, then, is $18 \cdot 8\pi = 144\pi \text{ in}^2$.

So, the sculpture’s faces have these areas in square inches:

- 2 bases: $48\sqrt{3} + 32\pi \text{ in}^2$
- 3 rectangular lateral faces: $144 \text{ in}^2$
- 1 curved lateral face: $144\pi \text{ in}^2$

The total surface area is $2(48\sqrt{3} + 32\pi) + 3(144) + 144\pi = 96\sqrt{3} + 208\pi + 432$, or about 1252 in$^2$.

If the top base of the sculpture were shaved to a point, the curved face would become half of a cone and the rest would become half of a hexagonal pyramid. You can use this question to motivate the study of the Pythagorean Theorem in Chapter 9. A student who remembers this theorem from a previous course can calculate the slant height of the cone to be $\sqrt{8^2 + 18^2} = \sqrt{388}$ and the slant height of the lateral faces of the half-pyramid to be $\sqrt{(4\sqrt{3})^2 + 18^2} = \sqrt{372}$. The surface area of a cone is $\pi rl + \pi r^2 = \pi r(\sqrt{388}) + \pi(8)^2$. Half of the cone would have half of this surface area, or $4\pi\sqrt{388} + 32\pi$. The lateral faces of the half-pyramid would have area $\frac{1}{2}bh = \frac{1}{2}(8)(\sqrt{372})$, or $4\sqrt{372}$. The bottom base would still have area $48\sqrt{3} + 32\pi$. Therefore, the total area of the figure would be $4\pi\sqrt{388} + 32\pi + 3(4\sqrt{372}) + 48\sqrt{3} + 32\pi$, which is about 763 in$^2$. 
Chapter 8 • Review Exercises

1. *(Lesson 8.1)* Find the area of quadrilateral $ABCD$ with vertices $(1, 2), (4, 2), (-1, -2), (2, -2)$.

2. *(Lessons 8.2, 8.4)* Find the area of each figure.
   a. Kite $ABCD$
   b. Triangle
   c. Regular hexagon

3. *(Lessons 8.1, 8.2, 8.3)* Shahin wants to carpet the floors in his living room and bedroom. If the carpet, padding, and installation cost $30 per square yard, how much will it cost?

4. *(Lessons 8.5, 8.6)* Find the area of the shaded region. Leave answers in terms of $\pi$.

5. *(Lesson 8.7)* Find the surface area of this square pyramid.
1. After plotting the points, we find the quadrilateral is a parallelogram with base equal to 3 units and height equal to 4 units. Find the area:

\[ A = bh \]

\[ A = (3)(4) \]

\[ A = 12 \text{ square units} \]

2. a. \[ A = \frac{1}{2}d_1d_2 \]

\[ A = \frac{1}{2}(10)(5) = 25 \text{ cm}^2 \]

b. \[ A = \frac{1}{2}bh \]

\[ A = \frac{1}{2}(19)(9) = 85.5 \text{ cm}^2 \]

c. \[ A = \frac{1}{2}aP \]

\[ P = 6 \cdot 20 = 120 \text{ in.} \]

\[ A = \frac{1}{2}(10\sqrt{3})(120) \approx 1039 \text{ in}^2 \]

3. Bedroom (trapezoid): \[ A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2} \cdot 10(10 + 20) = 150 \text{ ft}^2 \]

Living room (rectangle): \[ A = bh = (15)(20) = 300 \text{ ft}^2 \]

Total area = 150 + 300 = 450 ft²

To find the area in square yards:

\[ 450 \text{ ft}^2 \left( \frac{1 \text{ yd}^2}{3 \text{ ft}^2} \right) = 450 \text{ ft}^2 \cdot \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = 50 \text{ yd}^2 \]

Total cost = 50 yd² • \$30/yd² = $1500.00

4. \( r = 5 \text{ cm} \)

Angle measure of sector = \( 360^\circ - 120^\circ = 240^\circ \)

Area of a sector = \( \frac{240}{360} \cdot \pi r^2 = \frac{240}{360} \cdot \pi(5)^2 = \frac{50\pi}{3} \text{ cm}^2 \)

5. \[ A = 4\left(\frac{1}{2}bh\right) + b^2 \]

\[ A = 4\left(\frac{1}{2} \cdot 8 \cdot 9\right) + 8^2 = 208 \text{ in}^2 \]
CHAPTER 9

The Pythagorean Theorem

Content Summary

One of the best-remembered theorems of geometry is the Pythagorean Theorem. This chapter begins by reviewing the Pythagorean Theorem and then considering its converse. It then examines shortcuts in the case of special triangles. Finally, the Pythagorean Theorem is used to calculate distances on the coordinate plane, making it possible to write an equation for a circle.

The Pythagorean Theorem and Its Converse

Many people identify the Pythagorean Theorem as $a^2 + b^2 = c^2$, without recalling what $a$, $b$, and $c$ stand for. The theorem actually says that if $a$, $b$, and $c$ are the lengths of sides of a right triangle whose hypotenuse has length $c$, then $a^2 + b^2 = c^2$.

The converse of the Pythagorean Theorem is also true: If $a$, $b$, and $c$ are the lengths of sides of a triangle, and if $a^2 + b^2 = c^2$, then the triangle is a right triangle whose hypotenuse has length $c$.

Applications to Triangles

Discovering Geometry illustrates how to apply the Pythagorean Theorem to solve real-world problems. It also applies the theorem to find the relative lengths of sides of two special right triangles: the 45°-45°-90° triangle, which is half of a square; and the 30°-60°-90° triangle, which is half of an equilateral triangle.

Applications to Circles

From algebra, students know how to write equations to represent lines and parabolas. Coupled with the conjectures from Chapter 6, the Pythagorean Theorem can be used to develop equations for circles. The restatement of the Pythagorean Theorem in coordinate geometry is the distance formula, which tells how to find the distance between two points whose coordinates are known. From this formula, the book derives the equation of a circle.

\[ d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \]

(continued)
Chapter 9 • The Pythagorean Theorem (continued)

Summary Problem
Why is the general equation of a circle \((x - h)^2 + (y - k)^2 = r^2\)?

Questions you might ask in your role as student to your student:

- What numbers do \(h\), \(k\), and \(r\) represent?
- What points satisfy the equation if \(h\) and \(k\) are both 0?
- What can you say about the distance from any point on a circle to the center?
- How does this equation compare with the distance formula?
- Why does the sum of the squares on the left equal the square on the right?

Sample Answers
Encourage your student to relate this equation to the distance formula shown above. Looking at the diagram, imagine that the point \((x_1, y_1)\) is the center of a circle, and all the points on the circle are a distance \(d\) away from it. If you rename the center \((h, k)\) and rename the distance \(r\), you can see that the circle is all the points \((x, y)\) that are this distance away from \((h, k)\).

If the center is at the point \((0, 0)\), the points satisfying the equation are all a distance \(r\) from the origin—that is, they all lie on a circle centered at the origin and having radius \(r\).

The equation of that circle is \(x^2 + y^2 = r^2\), which is the Pythagorean Theorem. Try having your student use a compass to draw a circle centered at the origin with a certain radius (for example, 5 or 10), and then ask him or her to identify a few points on that circle and substitute them into the equation.
Chapter 9 • Review Exercises

Round solutions to the nearest tenth of a unit.

1. (Lesson 9.1) Find each missing length.

![Triangle diagram]

2. (Lesson 9.2) Is a triangle with sides measuring 3 ft., 4 ft., and 5 ft. a right triangle? Justify your answer.

3. (Lessons 9.3, 9.4) A 40-in. TV has a diagonal equal to 40 in., as shown here. Find the dimensions of the TV.

![TV diagram]

4. (Lesson 9.5) Find the equation of the circle with diameter endpoints (−2, 2) and (4, 2).

5. (Lesson 9.6) Find the area of the shaded region.

![Circle diagram]
1. \(a = 6\) cm
   Congruent segments.
   \[6^2 + b^2 = 19^2\]
   Pythagorean Theorem.
   \[36 + b^2 = 361\]
   \[b^2 = 325\]
   \[b \approx 18.0\] cm

2. If sides with lengths 3, 4, and 5 form a right triangle, the Pythagorean Theorem should make a true statement:
   \[3^2 + 4^2 = 5^2\]
   \[25 = 25\]
   Therefore, it is a right triangle.

3. Use the Triangle Sum Conjecture to find the other angle of the right triangle. The triangle is a 30°-60°-90° triangle, so you can use the shortcut.

```
   60°

   a

   40 in.

   b

   30°
```

   \[a = \frac{1}{2}(40) = 20\] in.
   \[b = a\sqrt{3} = 20\sqrt{3} \approx 34.6\] in.

4. Using the given diameter, the center is at \((1, 2)\), and the radius is 3. Enter these values into the circle equation.
   \[(x - 1)^2 + (y - 2)^2 = 9\]

5. The triangle is a 45°-45°-90° triangle, therefore the diameter is \(4\sqrt{2}\) m.
   \[r = \frac{1}{2}(4\sqrt{2}) = 2\sqrt{2}\] m
   Area of semicircle = \(\frac{1}{2}\) area of circle = \(\frac{1}{2}\pi r^2 = \frac{1}{2}\pi (2\sqrt{2})^2 = \frac{1}{2}\pi (4 \cdot 2) = 4\pi\) \(m^2\)
   Area of triangle = \(\frac{1}{2}bh = \frac{1}{2}(4)(4) = 8\) \(m^2\)
   Area of shaded region = area of semicircle − area of triangle = \(4\pi - 8 \approx 4.6\) \(m^2\)
CHAPTER 10

Volume

Content Summary

In geometry, the word *solid* refers to the shell of a three-dimensional figure, not including its interior. Solids can be classified in many ways, based on their shape. The five *regular* solids have faces that are congruent regular polygons and angles that are all congruent.

![Regular tetrahedron (4 faces)](image)

![Regular octahedron (8 faces)](image)

![Regular icosahedron (20 faces)](image)

![Regular hexahedron (6 faces)](image)

![Regular dodecahedron (12 faces)](image)

Just as area is measured as the number of unit squares that completely fill a region, volume is measured as the number of unit cubes that completely fill a space. They may be cubic inches (in³), cubic centimeters (cm³), or other units of volume.

This chapter contains formulas for the volume of quite a few solids. Your student should be able to recognize those formulas through visualization and a good understanding of volume, and use them to solve practical problems.

Prisms and Cylinders

Many common solids, such as cardboard boxes, have at least one pair of faces parallel and, usually, congruent. If these faces are polygons and are connected by sides that are parallelograms, the solid is called a *prism*. If the faces are circles joined by a single side, the solid is a *cylinder*. In either case, the parallel faces are called the *bases*.

To help your student think about the volume of a prism or cylinder, you might suggest imagining the prism on a table with one base downward. Then cover the bottom base with one layer of cubes. Some cubes may have to be cut to fit inside the solid. In any event, each of these cubes is providing a unit square on the base, so the number of these cubes is the area of the base. If you put another layer of cubes on top of the first, and then another layer on top of this, and so on, you can fill the prism. The volume—the number of cubes in the whole space—is equal to the number of cubes on the base times the number of layers. That is, the volume is the area of the base times the height of the solid.

Pyramids and Cones

Suppose you have a pyramid or a cone. The solid has one base and tapers to a point. If that base is a polygon, the pointed solid is a *pyramid*. If the base is a circle, the pointed solid is a *cone*. If you lay it on its base, each layer will be smaller than the layer below. It is interesting to compare the volume of a pointed solid with the volume of a prism or cylinder with the same base. Somewhat surprisingly, the ratio of their volumes is always 1 to 3. So, the volume of the pointed solid is \( \frac{1}{3} \) the volume of the prism or cylinder with the same base and height.
Chapter 10 • Volume (continued)

Spheres

*Discovering Geometry* asks students to find the volume of a sphere (actually, a hemisphere) by measuring the amount of water it will hold compared to a cylinder with the same radius and height. To help your student remember the formula, you might use a different comparison: Ask how the volume of a hemisphere relates to the volume of a cone with the same base and height. Clearly, the hemisphere has a larger volume. It turns out that the hemisphere has twice as much volume. If the sphere has radius $r$, then the area of the base (of the cone and hemisphere) is $\pi r^2$. The height of the hemisphere and the cone is also $r$, so the volume of the cone is $\frac{1}{3}(\pi r^2)(r)$, or $\frac{1}{3}\pi r^3$, therefore the volume of the hemisphere is $\frac{2}{3}\pi r^3$. The volume of the sphere, then, is twice that amount, or $\frac{4}{3}\pi r^3$.

To help remember the formula for the surface area of a sphere, you might suggest that your student imagine a piece of paper cut in a circle to match the base of the hemisphere. That paper would not be enough to cover the hemisphere itself. Interestingly, however, it would cover exactly half of it. So, the surface area of the sphere is 4 times the area of that circle. That is, the surface area is $4\pi r^2$.

**Summary Problem**

One treat at an ice-cream store has the shape of a cone with a hemisphere on top. The cone has height 22 cm. The diameter of the hemisphere is 13 cm. The treat comes packed tightly in a box shaped like a prism with square bases. What are the volumes of the box and the treat?

Questions you might ask in your role as student to your student:

- What are the dimensions (length, width, and height) of the packing box?
- What is the volume of the packing box?
- What would be the volume of the packing box if it were changed to a cylinder?
- What is the volume of the cone?
- What is the volume of the hemisphere?

**Sample Answers**

Let’s assume the treat is packed perfectly upright. (If the treat is packed tilted so that the tip of the cone slips into a corner, a slightly smaller box may be possible.) The dimensions of the base of the packing box must be the same as the diameter of the hemisphere, or 13 cm. The hemisphere has radius 6.5 cm, so the height of the packing box is 28.5 cm. Therefore, the volume of the packing box is $13 \cdot 13 \cdot 28.5$, or 4816.5 cm$^3$. If the packing box were a cylinder instead, the bases would be circles with radii 6.5 cm, so their areas would be $\pi(6.5)^2$, and the volume of the box would be $\pi(6.5)^2(28.5)$, or about 3783 cm$^3$.

The cone has a base of area $\pi(6.5)^2$ and it has height 22, so its volume is $\frac{1}{3}\pi(6.5)^2(22)$, or about 973.4 cm$^3$. The hemisphere has volume $\frac{1}{2}\frac{4}{3}\pi(6.5)^3$, which is about 575.2 cm$^3$. So, the volume of the treat is about 1549 cm$^3$. 
Chapter 10 • Review Exercises

1. (Lessons 10.1, 10.2) Use the triangular prism to answer the following:
   a. Name the bases of the prism.
   b. What is the area of one base?
   c. What is the height of the prism?
   d. Find the volume.

2. (Lessons 10.2, 10.3) Find the volume of each solid.
   a. Rectangular pyramid
   b. Cone
   c. Sphere

3. (Lessons 10.3, 10.4) A semicircular cylinder trough has a diameter of 3 ft and a length of 5 ft. If 1 cubic foot of water is about 7.5 gallons, how many gallons will the trough hold?

4. (Lesson 10.5) You drop a rock into a cylinder with a radius of 3 in., raising the water level by 1 in. What is the volume of the rock?

5. (Lesson 10.7) Find the volume and surface area of this hemisphere.
1. a. \( \triangle ABC \) and \( \triangle DEF \) are the bases.
   
b. \( A = \frac{1}{2}bh \)
   \[ A = \frac{1}{2}(6)(5) = 15 \text{ cm}^2 \]

c. \( H = 12 \text{ cm} \)

d. \( V = BH \) (\( B \) = area of base; \( H \) = height of prism)
   \[ V = (15)(12) = 180 \text{ cm}^3 \]

2. a. \( V = \frac{1}{3}BH \)
   \[ B = (3)(19) = 57 \text{ m}^2 \]
   \[ H = 12 \text{ m} \]
   \[ V = \frac{1}{3}(57)(12) = 228 \text{ m}^3 \]

   b. \( V = \frac{1}{3}BH \)
   \[ V = \frac{1}{3}(\pi r^2)H \]
   \[ V = \frac{1}{3}\pi(1)^2(3) = \pi \approx 3.1 \text{ cm}^3 \]

   c. \( V = \frac{4}{3}\pi r^3 \)
   \[ V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3} \approx 523.6 \text{ m}^3 \]

3. \( r = \frac{1}{2}(3) = 1.5 \text{ ft.} \)
   
   \( V = BH \), where the base is a semicircle and \( H \) is the length of the trough
   \[ V = \left(\frac{1}{2}\pi r^2\right)H \]
   \[ V = \frac{1}{2}\pi(1.5)^2(5) = 17.7 \text{ ft}^3 \]
   \[ 17.7 \text{ ft}^3 \cdot \frac{7.5 \text{ gal}}{1 \text{ ft}^3} \approx 133 \text{ gal} \]

4. This new “slice” of water is shaped like a cylinder with radius 3 and height 1. The volume of this cylinder is \( \pi r^2H = \pi(3)^2(1) = 9\pi \approx 28.3 \text{ in}^3 \).

5. Volume of hemisphere = \( \frac{1}{2} \) volume of sphere
   \[ V = \frac{1}{2}\left[\frac{4}{3}\pi r^3\right] \]
   \[ V = \frac{1}{2}\left[\frac{4}{3}\pi(10)^3\right] \approx 2094.4 \text{ in}^3 \]
   
   Surface area of hemisphere = \( \frac{1}{2} \) surface area of sphere + area of base
   \[ SA = \frac{1}{2}(4\pi r^2) + \pi r^2 \]
   \[ SA = \frac{1}{2}(4\pi(10)^2) + \pi(10)^2 = 300\pi \approx 942.5 \text{ in}^2 \]
CHAPTER 11

Similarity

Content Summary

One of the most important kinds of thinking in mathematics is proportional reasoning—applying what we know about a figure to larger or smaller figures. In Chapter 11, students have the opportunity to practice this kind of thinking by applying it to similar figures—figures that are enlargements (stretches) or reductions (shrinks) of each other. A transformation that stretches or shrinks a figure by the same factor in all directions is a dilation, and the factor is called a scale factor.

Similar Polygons

In everyday usage, “similar” means alike in some ways. But in geometry, similar means exactly the same shape (but not necessarily the same size). Two polygons are similar if their corresponding angles are congruent and the lengths of corresponding sides all have the same ratio. This ratio is called the scale factor. Ratios of lengths of other corresponding parts of similar triangles are equal to the scale factor. The book concentrates on similar triangles and—as with congruence—finds shortcuts for determining similarity. These shortcuts can be applied to calculate lengths that can’t be measured directly, such as the height of a flagpole.

Parallel Lines

If lines are drawn between two sides of a triangle and they are parallel to the third side, then each of the lines creates a new triangle that is similar to the original. Therefore, these lines divide proportionately the two sides that they intersect.

Area and Volume

Some of the most important and surprising results involve relationships among the areas or volumes of similar figures. Suppose you double the length and width of a rectangle to form a larger rectangle. You might expect the area to double, but in fact it quadruples, or increases by a factor of 4.

If any two-dimensional figure is dilated by a scale factor of $r$, then its area is multiplied by $r^2$. Similarly, if a three-dimensional figure is dilated by a factor of $r$, then its volume is multiplied by $r^3$. (Surface areas of its faces, which are two-dimensional, are multiplied by $r^2$.) For example, if a sphere is enlarged by a factor of 1.5 (its radius is multiplied by 1.5), then its volume is multiplied by $1.5^3 = 3.375$, and its surface area is multiplied by $1.5^2 = 2.25$. (continued)
Chapter 11 • Similarity (continued)

Summary Problem

Draw a pentagon on graph paper, as in Lesson 11.1, Investigation 2. Choose any constant as your scale factor. Multiply the coordinates of each vertex by that constant to locate the vertices of a new pentagon. How do the two pentagons compare?

Questions you might ask in your role as student to your student:

● Are the pentagons similar?
● How many ways can you prove that they’re similar?
● How does the area of the new pentagon compare to the area of the original?
● What scale factors would you use to enlarge the original pentagon? What scale factors would you use to make a smaller pentagon?

Sample Answers

To verify similarity, we have to show that the corresponding sides are proportional and that the corresponding angles are congruent. Your student can verify that the sides are proportional either by measuring the lengths of corresponding sides and writing their ratios or by drawing segments from the origin to the vertices and applying the Extended Parallel/Proportionality Conjecture (finding five pairs of similar triangles). He or she can verify the congruence of corresponding angles by measuring or by applying the SSS similarity shortcut to triangles into which the pentagon can be divided. Once the scale factor is known, it can be squared to determine the ratio of areas. Scale factors larger than 1 will enlarge the figure, whereas scale factors smaller than 1 will reduce the size.
Chapter 11 • Review Exercises

Round your solutions to the nearest tenth of a unit unless otherwise stated.

1. (Lesson 11.1) \(ABCD \sim FGHI\). Find \(a\) and \(b\).

2. (Lesson 11.2) Which similarity shortcut can be used to show that \(\triangle ABC \sim \triangle EBD\)? Find \(a\) and \(b\).

3. (Lesson 11.3) Rafael is 1.8 m tall and casts a 0.5 m shadow. If at the same time a nearby tree casts a 1.5 m shadow, how tall is the tree?

4. (Lesson 11.4) Find \(x\).

5. (Lesson 11.5) \(ABCD \sim EFGH\).
   Area of \(ABCD = 15\) cm\(^2\)
   Area of \(EFGH = ?\)

6. (Lesson 11.6) Small triangular prism \(\sim\) Large triangular prism
   Volume of small prism = 48 in\(^3\)
   Volume of large prism = ?

7. (Lesson 11.7) \(EB \parallel CD\). Find \(a\).
1. Because the quadrilaterals are similar, corresponding angles are congruent. Therefore, \( a = m \angle G = m \angle B = 110^\circ \).

Because the two quadrilaterals are similar, we can set up a proportion to find the missing length.

\[
\frac{12}{7} = \frac{18}{b}
\]

Multiply both sides by 7; multiply both sides by \( b \).

\( 13b = 18 \cdot 7 \)  

\( b \approx 9.7 \text{ m} \)  

Solve for \( b \).

2. Two pairs of corresponding sides are proportional \((\frac{13.5}{4.5} = \frac{9}{3})\) and their included angles are congruent (vertical angles \( \angle ABC \) and \( \angle EBD \)), so the triangles are similar because of the SAS Similarity Conjecture. To find \( a \), use the Triangle Sum Conjecture to find \( \angle ABC: 180^\circ - 38^\circ - 111^\circ = 31^\circ \). Because \( \angle ABC \equiv \angle EBD \), \( a = m \angle EBD = 31^\circ \). To find \( b \), solve the proportion.

\[
\frac{9}{3} = \frac{7.5}{b}
\]

Multiply both sides by 3; multiply both sides by \( b \).

\( 9b = 7.5 \cdot 3 \)  

\( b \approx 2.5 \text{ cm} \)  

Solve for \( b \).

3. Because the sun’s rays all come in at the same angle at a particular time of day, all people or objects and their shadows form similar right triangles. So, Rafael’s height and the length of his shadow are proportional to the tree’s height and the length of its shadow.

\[
\frac{1.8}{0.5} = \frac{x}{1.5}
\]

Multiply both sides by 1.5.

\( x = 5.4 \)  

The tree is 5.4 m tall.

4. The angle bisector divides the opposite side into segments that are proportional to the other two sides of the triangle. Set up a proportion and solve:

\[
\frac{x}{15} = \frac{4}{9}
\]

Multiply both sides by 15; multiply both sides by 9.

\( x \approx 6.7 \text{ cm} \)

5. When two figures are similar, the ratio of their areas is the square of the scale factor.

\[
\frac{AD}{EH} = \frac{3}{7}
\]

The scale factor is the ratio of corresponding sides.

\[
\text{Area of } ABCD = \left(\frac{3}{7}\right)^2 = \frac{9}{49}
\]

The ratio of areas is the square of the scale factor.

\[
\frac{15}{x} = \frac{9}{49}
\]

Substitute values.

\( x \approx 81.7 \text{ cm}^2 \)  

Solve for \( x \).

6. Scale factor \( = \frac{6}{11} \)

Find the scale factor.

\[
\frac{V_{\text{small prism}}}{V_{\text{large prism}}} = \left(\frac{6}{11}\right)^3
\]

The ratio of the volumes is equal to the cube of the scale factor.

\[
\frac{48}{x} = \frac{216}{1331}
\]

Substitute values.

\( x \approx 295.8 \text{ in}^3 \)  

Solve for \( x \).

7. Because of parallel proportionality, corresponding sides are proportional.

\[
\frac{AB}{AC} = \frac{EB}{DC}
\]

\[
\frac{12}{12 + a} = \frac{16}{23}
\]

Substitute values.

\( 12 \cdot 23 = 16(12 + a) \)

Multiply both sides by 23\((12 + a)\).

\( 276 = 192 + 16a \)

Distribute.

\( a = 5.25 \)  

Solve for \( a \).
CHAPTER 12

Trigonometry

Content Summary
If two triangles are similar, then their corresponding sides have the same ratio of lengths. Say, \( \frac{a}{a'} = \frac{b}{b'} \). But the ratios of the corresponding side lengths of each triangle are also equal: \( \frac{a}{b} = \frac{a'}{b'} \).

Now, if two right triangles have a second angle congruent (in addition to their right angles), then the third angles will also be congruent (by the Triangle Sum Conjecture), so the triangles will be similar (by AAA). Therefore, ratios of the corresponding side lengths of each triangle are also equal. In other words, all right triangles with a certain acute angle measure, say 35°, are similar to one another. So, associated with any acute angle are various ratios of the side lengths, which are the same for all right triangles containing this angle.

Trigonometric Ratios
The study of the relationships between sides and angles of triangles is called trigonometry. The ratio of the lengths of two sides in a right triangle is called a trigonometric ratio. The three most commonly used ratios—sine, cosine, and tangent—are the focus of this chapter. The sine (abbreviated sin) of an angle is the ratio of the length of the opposite leg to the length of the hypotenuse. For example, the sine of 30° is \( \frac{1}{2} \). This means that in any right triangle with a 30° angle, the side opposite the 30° angle is \( \frac{1}{2} \) the length of the hypotenuse. The cosine (abbreviated cos) of an angle is the ratio of the adjacent leg to the hypotenuse. The tangent (abbreviated tan) is the ratio of the opposite leg to the adjacent leg. The trigonometric ratios for various angle measures can be looked up in a trigonometric table. They can also be found using a calculator with sin, cos, and tan keys. For example, sin 30 will give 0.5, as long as the calculator is in degree mode. Some problems require the inverse of the sine, cosine, or tangent—the ratio of lengths is known and the angle is needed. The inverse can also be found using a calculator. For example, sin\(^{-1}\) 0.5 = 30.

This chapter focuses primarily on right triangles, but any triangle can always be divided into two right triangles. Therefore, trigonometry also helps us understand other kinds of triangles and provides shortcuts for working with non-right triangles as well. The Law of Sines, the Law of Cosines, and the SAS Area Conjecture apply to all triangles.

(continued)
Chapter 12 • Trigonometry (continued)

Summary Problem

How can you find the area of a triangular plot of land whose sides have lengths 5 km, 8 km, and 9 km?

Questions you might ask in your role as student to your student:

- What do you know about areas of triangles?
- What would you need to know to apply the formula involving base and altitude?
- How could you determine those quantities?

Sample Answers

Your student may or may not have seen Hero’s formula in the Exploration Alternative Area Formulas in Chapter 8. That formula yields an area of about 19.9 km². More likely, students will think of the standard formula for the area of a triangle, \( A = \frac{1}{2}bh \). However, the height needs to be measured perpendicular to a base. Any side can serve as a base; to find the altitude to that side, students need an angle. They can use the Law of Cosines to find an angle at one end of the chosen base. Then they can apply the SAS Triangle Area Conjecture to find the area. For example, if the base is the side of length 9 km, then the angle \( x \) between that side and the side of length 8 km can be found using the Law of Cosines:

\[
5^2 = 9^2 + 8^2 - 2(9)(8)\cos x
\]

Substitute values into the Law of Cosines.

\[
\frac{25 - 81 - 64}{-144} = \cos x
\]

Solve for \( \cos x \).

\[
x = \cos^{-1}\left(\frac{5}{6}\right)
\]

Use the inverse cosine to find \( x \).

\[
x \approx 33.6°
\]

The sine of this angle is the ratio of the altitude to the side with length 8 km. The altitude then is 8 times the sine of this angle, or about 4.42 km. The area is \( 0.5(4.42)(9) \), approximately 19.9 km².
Chapter 12 • Review Exercises

1. (Lessons 12.1, 12.2) Find the missing measures to the nearest tenth of a unit.
   a. \[ \triangle x \quad 62^\circ \quad y \]
   b. \[ \triangle x \quad 22 \text{ cm} \quad y \]

2. (Lessons 12.3, 12.4) Find the missing lengths to the nearest centimeter.
   a. \[ \triangle 32 \text{ cm} \quad 95^\circ \quad x \]
   b. \[ \triangle 52 \text{ cm} \quad 15^\circ \quad y \]

3. (Lesson 12.5) Radha is flying her plane on a heading, as shown. Her air speed is 125 mi/h. There is a crosswind of 15 mi/h. What is her resulting speed \( T \)?
3. The diagonal of the parallelogram is the resulting speed. You can use the Law of Cosines, but you need to know at least one angle measure in the shaded triangle. Because the diagram is a parallelogram, the adjacent angles are supplementary.

![Diagram of a parallelogram with labeled sides and angles.]

Therefore, $65^\circ + x = 180$, so $x = 115^\circ$. Now you can use the Law of Cosines to find $r$.

$$r^2 = 15^2 + 125^2 - 2(15)(125)\cos 115^\circ$$

$$r^2 \approx 17,434.8$$

$$r \approx 132\text{ mi/h}$$
CHAPTER 13

Geometry as a Mathematical System

Content Summary
Having experienced all the concepts of a standard geometry course, students are ready to examine the framework of the geometry knowledge they have built. Students now review and deepen their understanding of those concepts by proving some of the most important conjectures in the context of a logical system, starting with the premises of geometry.

Premises and Theorems
A complete deductive system must begin with some assumptions that are clearly stated and, ideally, so obvious that they need no defense. Chapter 13 begins by laying out its assumptions: properties of arithmetic and equality, postulates of geometry, and a definition of congruence for angles and line segments. These basic assumptions are called premises. Everything else builds on these premises.

Next, students develop proofs of their conjectures concerning triangles, quadrilaterals, circles, similarity, and coordinate geometry. Once a conjecture has been proved, it is called a theorem. Each step of a proof must be supported by a premise or a previously proved theorem.

Developing a Proof
Developing a proof is more art than science. Mathematicians don’t sit down and write a proof from beginning to end, so encourage your student not to expect to do so. Proofs require thought and creativity. Generally, the student will start by writing down what’s given and what’s to be shown—the beginning and end of the proof. Then, perhaps using diagrams, the student will restate these first and last statements in several ways, looking for an idea of how to get from one to the other logically. You might remind your student about the reasoning strategies that can help in planning a proof:

- Draw a labeled diagram and mark what you know
- Represent a situation algebraically
- Apply previous conjectures and definitions
- Break a problem into parts
- Add an auxiliary line
- Think backward

Questions you may ask your student include “What can you conclude from the given statements?” and “What’s needed to prove the last statement?” Seeing connections, the student can develop a plan, perhaps expressed using flowcharts. There are several ways to express the plan, and your student may use more than one. Then he or she can write the proof, being careful to check the reasoning. A good way to be careful about details is to write a two-column proof, with statements in the first column and reasons in the second. Your student might find gaps in his or her reasoning and have to go back to the planning stage.

If there doesn’t seem to be any way to prove the statement, you might suggest indirect reasoning, in which the student proves that the negation of the theorem is false. It then follows that the theorem must be true.

(continued)
Chapter 13 • Geometry as a Mathematical System (continued)

Summary Problem

Draw a diagram that gives a family tree for all triangle theorems that appear in the exercises for Lesson 13.3. Include postulates and theorems, but not definitions or properties.

Questions you might ask in your role as student to your student:

- Can you build on the tree on page 707?
- What do the arrows represent in words?

Sample Answers

A family tree shows how theorems support each other. A theorem may rely on several theorems, each of which relies on other theorems, and so on, all the way up to the postulates of geometry. The top of the family tree should be all postulates, and all the arrows should flow down from there. The diagrams can get quite complex. Don’t worry too much about neatness or completeness; the goal is to see how the structure can be built while reviewing the theorems. Here is the complete tree.

![Family Tree Diagram]

SAS Postulate

ASA Postulate

Parallel Postulate

CA Postulate

Linear Pair Postulate

Angle Addition Postulate

SSS Postulate

Perpendicular Bisector Theorem

Isosceles Triangle Theorem

Perpendicular Bisector Theorem

Linear Pair

Postulate

Parallel Postulate

CA Postulate

Linear Pair Postulate

Angle Addition Postulate

SSS Postulate

VA Theorem

AIA Theorem

Triangle Sum Theorem

Third Angle Theorem

SAA Congruence Theorem

Converse of Isosceles Triangle Theorem

Converse of Angle Bisector Theorem

Angle Bisector Theorem

Angle Bisector Theorem

Angle Bisector Concurrency Theorem

Altitudes to Congruent Sides Theorem

Medians to Congruent Sides Theorem

Perpendicular Bisector Concurrency Theorem

Converse of Perpendicular Bisector Theorem
Chapter 13 • Review Exercises

1. (Lesson 13.1) Name the property that supports each statement:
   a. If \( \overline{AB} \cong \overline{CD} \) and \( \overline{CD} \cong \overline{EF} \), then \( \overline{AB} \cong \overline{EF} \).
   b. If \( \overline{AB} \cong \overline{CD} \), then \( AB = CD \).

2. (Lessons 13.2, 13.3) In Lesson 13.2, Example B, the Triangle Sum Theorem is proved with a flowchart proof. Rewrite this proof using a two-column proof.
   Given: \( \angle 1, \angle 2, \) and \( \angle 3 \) are the three angles of \( \triangle ABC \)
   Show: \( m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ \)

3. (Lessons 13.2, 13.4) Answer the following questions for the statement, “The diagonals of an isosceles trapezoid are congruent.”
   a. Task 1: Identify what is given and what you must show.
   b. Task 2: Draw and label a diagram to illustrate the given information.
   c. Task 3: Restate what is given and what you must show in terms of your diagram.

4. (Lesson 13.6) Write a proof for the Parallel Secants Congruent Arcs Theorem: Parallel lines intercept congruent arcs on a circle.

5. (Lesson 13.7) Write a proof for the Corresponding Altitudes Theorem: If two triangles are similar, then corresponding altitudes are proportional to the corresponding sides.
1. a. Transitive Property  
b. Definition of Congruence

2.

3. a. Given: Isosceles trapezoid  
   Show: Diagonals are congruent  
   b.

4. 

Given: \( \overline{AB} \parallel \overline{DC} \)  
Show: \( \overline{AD} \equiv \overline{BC} \)
5. 

Given: $\triangle CBD \sim \triangle JKL$; Altitudes $\overline{CP}$ and $\overline{JR}$

Show: $\frac{CP}{JR} = \frac{CB}{JK}$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle CBD \sim \triangle JKL$; Altitudes $\overline{CP}$ and $\overline{JR}$</td>
<td>Given</td>
</tr>
<tr>
<td>$\overline{CP} \perp \overline{BD}$; $\overline{JR} \perp \overline{KL}$</td>
<td>Definition of Altitude</td>
</tr>
<tr>
<td>$\angle CPB$ and $\angle JRK$ are right angles</td>
<td>Definition of Perpendicular</td>
</tr>
<tr>
<td>$\angle CPB \cong \angle JRK$</td>
<td>Right Angles are Congruent Theorem</td>
</tr>
<tr>
<td>$\angle CBD \cong \angle JKL$</td>
<td>Corresponding Angles of Similar Triangles are Congruent</td>
</tr>
<tr>
<td>$\triangle CBP \sim \triangle JKR$</td>
<td>AA Similarity Postulate</td>
</tr>
<tr>
<td>$\frac{CP}{JR} = \frac{CB}{JK}$</td>
<td>Corresponding Sides of Similar Triangles are Proportional</td>
</tr>
</tbody>
</table>